Computer assisted investigation in the teaching of mathematics

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Abstract
Procedures based on experimentation and discovery play an important role in mathematical education. Computer Algebra and Dynamic Geometry Systems are suitable environments for the implementation of activities based on these procedures. The presentation brings several specific examples of various complexity that come from school practice. These examples on different topics, from financial mathematics to curves and surfaces, show how the use of CAS and DGS, jointly or separately, can facilitate the understanding of the relevant mathematical phenomena or property, or the solving of a given mathematical problem.

1. Introduction

Objective of the paper is to present different ways of the utilization of CAS and DGS in the mathematics teacher training courses at the Faculty of Education of the University of South Bohemia. We introduce the reader to the role of computers in mathematics teacher training, acquaint him or her with examples of particular applications of CAS and DGS in teacher training courses and show him or her some selected examples taken from these courses.

2. Mathematics teacher training

The fundamental task of the teacher training courses is to prepare and promote the further development of a competent teacher. To ensure the optimum development of the relevant content knowledge of such a teacher the authors follow Shulman's three categories: subject matter content knowledge, pedagogical content knowledge and curricular knowledge (Shulman 1986). A deep understanding of relevant mathematical issues and their position in the curriculum together with the knowledge of how to teach them allow the teacher to develop pupils' knowledge in an effective way, in accordance with the current state of knowledge and the abilities of pupils.

The faculty of Education pays attention to the cultivation of all three categories in both pre-service and in-service teacher training courses. The effective use of computers to support preparation of mathematics teachers is a subject of constant study. In addition to the regular use of computers in the teaching of university courses, the issues of the utilization of computers in mathematics education are dealt with in several research or practically oriented projects and the final works of students. These are also the main topics of regular conferences and meetings with in-service teachers that the faculty organises.

2. 1. Mathematics content knowledge

The development of the mathematics content knowledge of future teachers is included in the university mathematical courses. The most commonly used computer programmes are Maple, wxMaxima, GeoGebra, Cabri 3D and CoCoA.

For example, the computer algebra system Maple (www.maplesoft.com) is used to support the teaching of regular higher-level mathematics courses (see for example Hašek and Pech 2010) and it also proved to be an effective tool for the financial education of future

teachers at the faculty. In particular advantage is taken of the so called Maple "smart documents", as has been depicted for example in Hašek and Petrášková (2010) or Petrášková and Hašek (2012).

In the category of DGS the software GeoGebra (www.geogebra.org) is clearly the most used, particularly in calculus and geometry courses. In the teaching of constructive geometry Cabri3D (www.cabri.com) has its obvious place.

2. 2. Pedagogical content knowledge

To promote the development of the pedagogical content knowledge of future teachers the members of the department of mathematics of the Faculty of Education together with students systematically pay attention to finding effective ways for the use of computers in mathematics education. This clear aim is reflected in the diploma, doctoral thesis (Suchopárová 2010, Štrausová 2014) and seminar works of students. Conferences and research projects that the department organizes or is involved in are also focused on this main research target.

The use of computers in schools should reflect the differences between the curricula of basic and secondary schools. Whereas the basic school mathematics is focused mainly on the development of the pupil's geometrical vision and calculation skills, the secondary school mathematics should promote abstraction, logical thinking and argumentation skills.

To cultivate the geometrical vision of basic school pupils we can apply GeoGebra as shown in fig. 1. Its function to insert an image into Graphics allows a pupil to study geometrical shapes or transformations on pre-prepared figures.

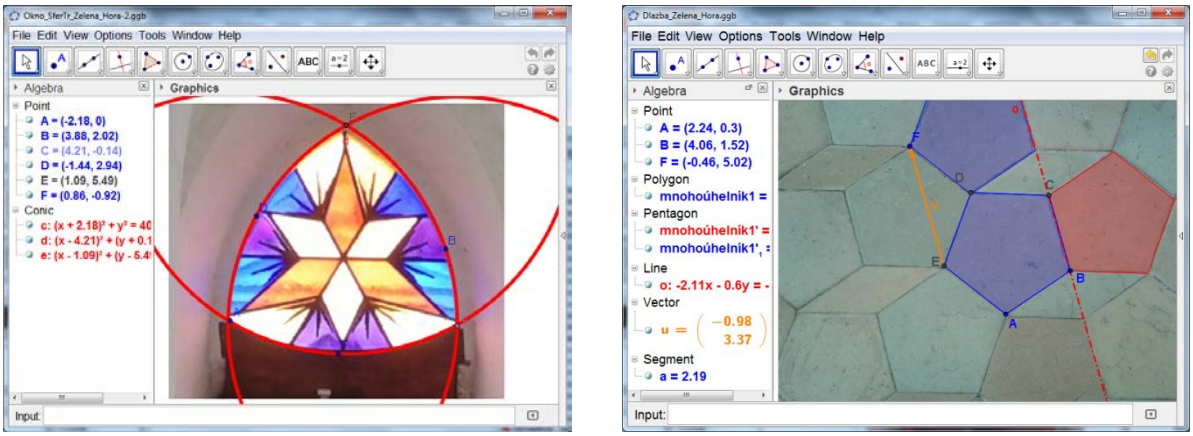

Figure 1: Geometry around us (Photographed by the author in the Church of St. John of Nepomuk at Zelena hora, www.zelena-hora.eu/en/)

GeoGebra provides a user with different environments that work with common data and can be used separately or jointly. For example, joint use of Graphics and Spreadsheet allows us to record changes in the parameters of Graphics into Spreadsheet and re-display the collected data graphically. As shown in fig. 2 we can use this method to investigate the relation between the perimeter and diameter of a circle.

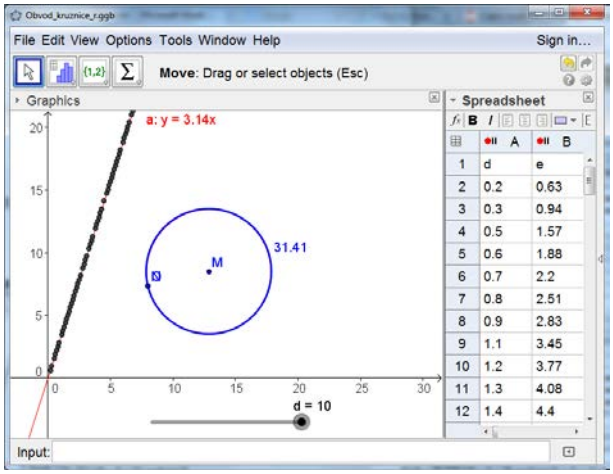

Figure 2: Perimeter vs. Diameter

The use of the real pantograph has been well proven with pupils. They like to use it to change the size of a picture and they naturally wish to learn something about its principle. To support their investigation of this principle the creation of a model in GeoGebra (see fig. 3) is beneficial.

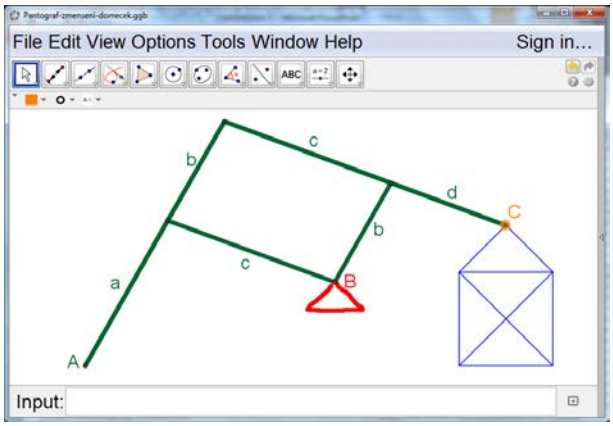

Figure 3: Pantograph modelled in GeoGebra

At secondary school, where proofs should have their place in mathematics to develop the argumentation skills of students, by means of using the computer we can solve the classical so called Regiomontanus' problem (Dorrie 1965) with students (see fig. 4): From the foot, at what distance does the statue appear highest?
In addition to the geometrical solution, that is based on the inscribed angle theorem, GeoGebra provides us with the possibility to record the corresponding data on Spreadsheet, display them in Graphics and find the solution to this extreme problem by inspecting this graph.

Figure 4: Regiomontanus' problem

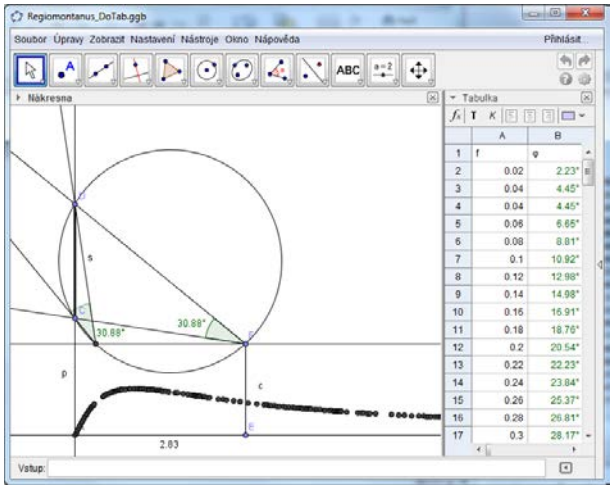

Figure 5: Regiomontanus' problem solved in GeoGebra

2. 3. Curricular Knowledge

To develop the curricular knowledge of future mathematics teachers it is necessary to be in contact with practice. Good opportunities for such contact and for the sharing of teacher created educational materials are national and international projects. One example was the Czech-Austrian project Matematech (www.MatemaTech.cz) that focused on ways to increase student interest in mathematics. Materials collected by Czech and Austrian secondary school teacher participants of the project are published on the web page of the project and a selection of them were published in the bilingual book (Hašek (ed.) 2014).

3. Conclusion

As has been proven through many researches the use of computers in mathematics education can be beneficial. The teacher is the main factor that has an impact on it. Therefore the right preparation of teachers plays a crucial role in the process of the effective implementation of computers into mathematics teaching.

References

Dorrie, H. (1965) *100 Great Problems of Elementary Mathematics*. New York: Dover Publications, Inc.

Hašek, R., Pech, P. (2010) *Kvadratické plochy a jejich reprezentace v programu Maple*, České Budějovice: Pedagogická fakulta JU v Č. B. Available at http://www.pf.jcu.cz/stru/katedry/m/knihy/Analyticka.pdf

Hašek, R. and Petrášková, V. (2010) Issue of Financial Capability. *The International Journal for Technology in Mathematics Education*. Research Information Ltd., Burnham, UK. Volume 17, Number 4. pp. 183-190.

Hašek, R. (ed.) (2014) *MatemaTech – Matematika přes hranice, MatemaTech – Mathematik über Grenzen*, Jihočeská univerzita v Českých Budějovicích. Available at http://www.matematech.cz/wp-content/uploads/2014/09/Matematech-Matematika-pres-hranice-odk.pdf

Petrášková, V., Hašek, R. (2012) Financial education demands concerning teacher training. *Acta Didactica Universitatis Comenianae, Mathematics*, Issue 12, 2012, pp. 65 – 82. Available at http://www.ddm.fmph.uniba.sk/ADUC

Shulman, L. S. (1986) Those Who Understand: Knowledge Growth in Teaching, *Educational Researcher*, Vol. 15, No. 2. (Feb., 1986), pp. 4-14.

Suchopárová, T. (2014) *Archimedes' book of lemmas* [online]. Available at http://www.geogebratube.org/student/b135598

Štrausová, I. (2014) *Dynamic proofs* [online]. GeoGebraTube web portal. Available at http://www.geogebratube.org/student/b73724