Cognitive-visual approach to the teaching topic "Derivative of a function"

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Introduction

Research in cognitive psychology indicates that our brains store knowledge using both symbols and images. The brain is the only organ that expresses functional asymmetry (**lateralization**).

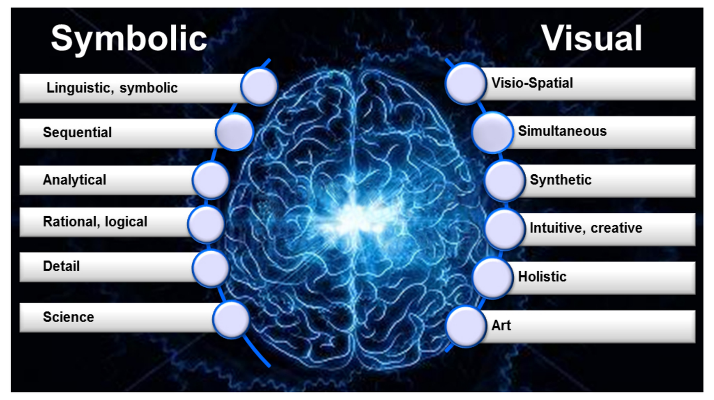


Figure 1: Lateralization of brain function

The mental processes involve both cerebral hemispheres, with one hemisphere dominant for certain functions or different aspects of the same functions.
In Figure1 hemispheric specialization of functions and ways of processing information are presented.

The findings of cognitive psychologists have an impact on research in mathematics education. So Tall and Winner In their work two ways of adopting mathematical concepts are stated: the **concept definition** and the **concept image**. The concept definition is a form of words and symbols used to specify that concept. The term concept image is defined as "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes". During the formation of the concept, the relationship between the concept definition and the concept image should be reciprocal [1].

Figure 2: Relation between the two concepts during concept formation is reciprocal

Pedagogical research indicates that in teaching practice, depending on the content being taught, we should find an optimal relationship between the concept of definitions and the concept of images, between the logical-analytical and visual creative thinking. Teaching and learning should be based on a balanced functioning of both hemispheres.

Cognitive-visual approach to teaching calculus

Functions first derivative is one of the basic topics of calculus. Students have difficulties accepting the concepts of calculus, because besides the great knowledge of basic math, must form a **"advanced mathematical thinking"** (thinking which is based on formal definitions, axioms and theorems, with which, logical deduction is applied).

Visualizing concepts and processes of calculus that are being introduced or processed in combination with symbolic entries and definitions contribute to the efficiency of the educational process. The essence of cognitive-visual approach to the design of a learning environment in which **cognitive graphics** associated geometric and symbolic representations of mathematical concepts and processes, and the associated cognitive processes.

Realization of the cognitive-visual approach means that we apply graphics, schemes, concept maps, interactive learning materials, applets, animations, visualized problems etc. in teaching process. By applying according computer programs, the teacher can realize the teaching process in visual environment. One of the packages that are used in teaching is GeoGebra.

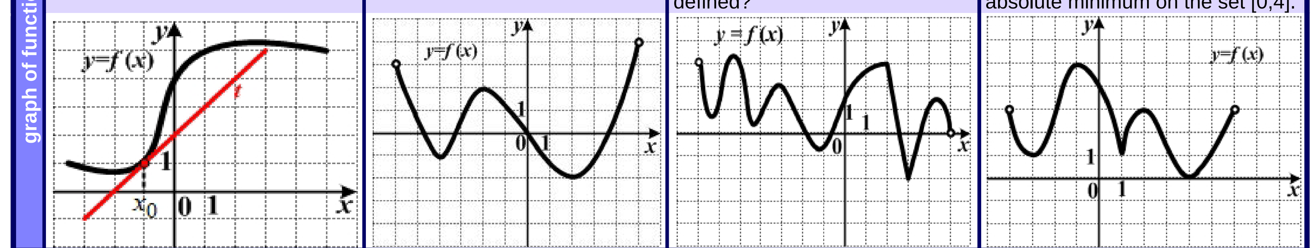
GeoGebra is a dynamic mathematical software where mathematical objects are shown in two ways: algebraic and graphic. This tool extends the concepts of dynamic geometry to the fields of algebra and mathematical analysis.

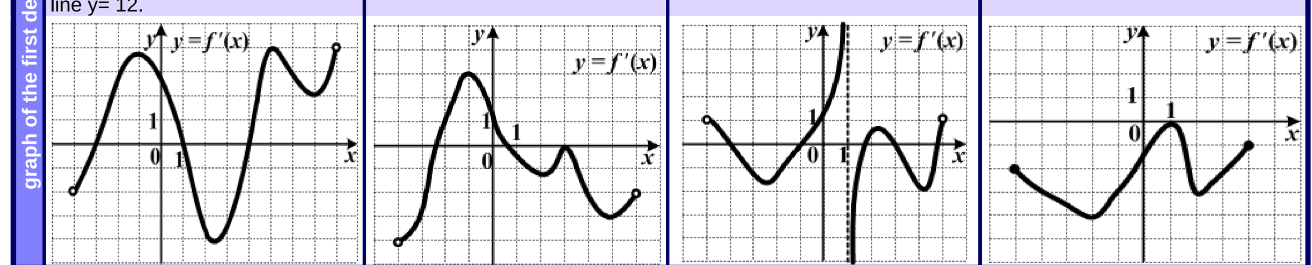
Visualized problems in the teaching topic "Derivative of a function"

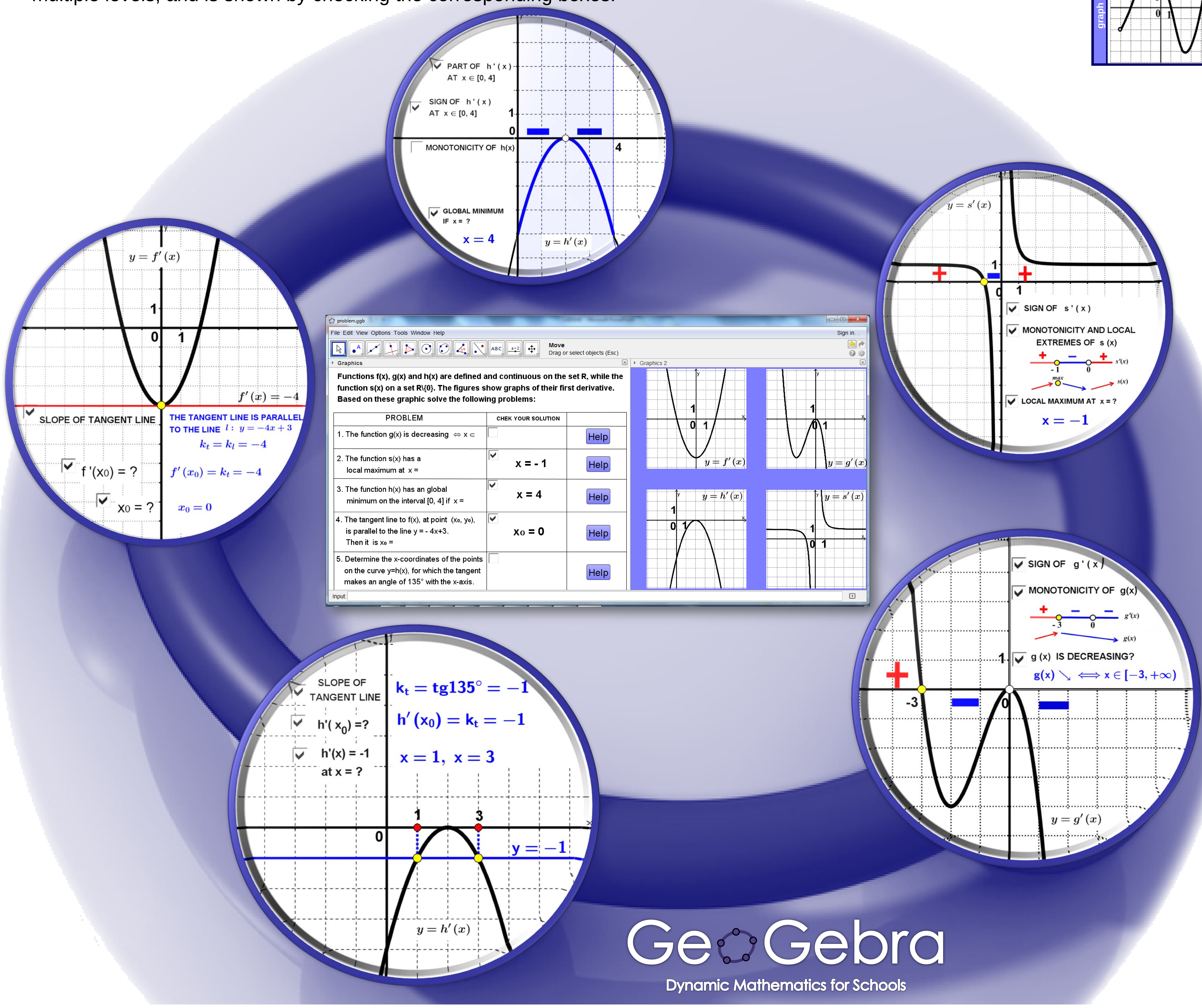
One of the possibilities to implement cognitive-visual approach in calculus are **visualized problems**. These are problems in which the image is explicitly or implicitly included in the very manner the problem is formulated, in the way of solving the problem or in the final solution [2]. Figure 3 shows examples of such problems. Function graph (Derivative's graph) is displayed in the grid, so that the image contains the data required for the solution. If the figure shows a function graph, then the first derivative is discussed and vice versa. To solve the problem, it is necessary both to apply the visual understanding and thinking and the knowledge of various mathematical fields.

We are presenting examples of how **GeoGebra** is used in classrooms with students, to explain and explore concept of first derivative. The basic idea while preparing GeoGebra dynamic worksheets is connecting algebraic and geometrical interpretation of the concept of derivative of a function.
Dynamic worksheets allow students to verify if they solved the problem correctly (check your solution). In case that students haven't solved the assignment correctly, or if during solving they have certain difficulties and concerns, they can use additional explanations by clicking the "Help" button. The assistance that students can receive is organized in multiple levels, and is shown by checking the corresponding boxes.

	geometric interpretation of the first derivative	monotonicity of the function	local extremes	absolute extremes on the set
graph of function	The figure shows the part of the function graph y=f(x) and tangent graphics at the point (x_0, y_0). Calculate the value of the expression f '(x_0) + f(0) - f(1).	The figure shows the function graph y=f(x) which is defined on the interval (-6, 5). Calculate the sum of all integers for which the first derivative is positive.	The figure shows the function graph y=f(x) which is defined on the interval (-7, 5). At what points of the local extremes of function y=f(x), is first derivative not defined?	The figure shows the function graph y=f(x) which is defined on the interval (-4, 6). Determine the value of first derivative at the point where the function y=f(x) has the absolute minimum on the set [0,4].
graph of the first derivative	Function y=f(x) is defined on the interval (-4, 8). The figure shows the first derivative graph y=f'(x). Calculate the sum of the abscissa points in which the tangents of the function y=f(x) are parallel to the line y= 12.	Function y=f(x) is defined on the interval (-4, 6). The figure shows the first derivative graph y=f'(x). How many integers belong to intervals where the function is decreasing?	Function y=f(x) is defined and continuous on the interval (-5, 5). The figure shows the first derivative graph y=f'(x). At how many points of function y=f(x) does it have the local maximum?	Function y=f(x) is defined on the interval [-5, 4]. The figure shows the first derivative graph y=f'(x). For what value of the argument x the function y=f(x) has the absolute minimum?







Findings

In the course of the teaching process the authors have used visualized problems and GeoGebra applets, with students in high school and faculty.

- After a short training, students can solve a number of problems in a short period of time.
- Solving the problem students experienced as a game with images and were very motivated.
- Students were satisfied because they had the option to create the way by which they get to the final solution, by themselves.
- These students have shown better results than the students from the previous years, when the cognitive-visual approach was not applied.

Conclusion

The role of the teacher is very important in the transition process of students from elementary to advanced mathematical thinking in the learning of calculus.
Problems visualized in this way enable the development and the use of visual thinking and the accomplishment of functional interdependence of graphic and analytical forms.
Visual and dynamic interaction between the user and GeoGebra environment helps students to form their visual understanding and connecting with the formal-symbolic language of calculus.

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