

**CADGME 2014**

An approach to the study of systems of equations with  
GeoGebra: learning opportunities provided by the  
integration of CAS view



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CENTRO DE  
ESTUDIOS EN  
DIDÁCTICAS  
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## The point of depart:

**What's happen with the instruction situation in relation with the topic "equations systems"?**

In Argentina, solve equations systems at school is usually a task for students who present in a series of techniques that "allow" find the solution.

Generally, these techniques work as an end in themselves. Finally and as a strategy of "verification" tends to give a "new resolution method": graph.

This way of conceiving mathematical practice fragments the possibilities for students to build a global view of the subject that hinders the recognition of the complementarity of the different ways to represent a mathematical notion.

## Initials questions:

- How to overcome educational obstacles that are generated from a fragmented approach of knowledge?
- What can bring the DGS, in particular the CAS environment?
- What epistemic and instrumental value (Artigue 2002) acquires the techniques work with software? Redefine work with pencil and paper?

## GeoGebra and the equations system:

This presentation will work on a series of dynamic problems (Bifano & Villella, 2012) to study the intersection of equations manipulating different parts of the same. These problems, which can be solved with pencil and paper, open a number of issues of educational interest when resolved with GeoGebra.



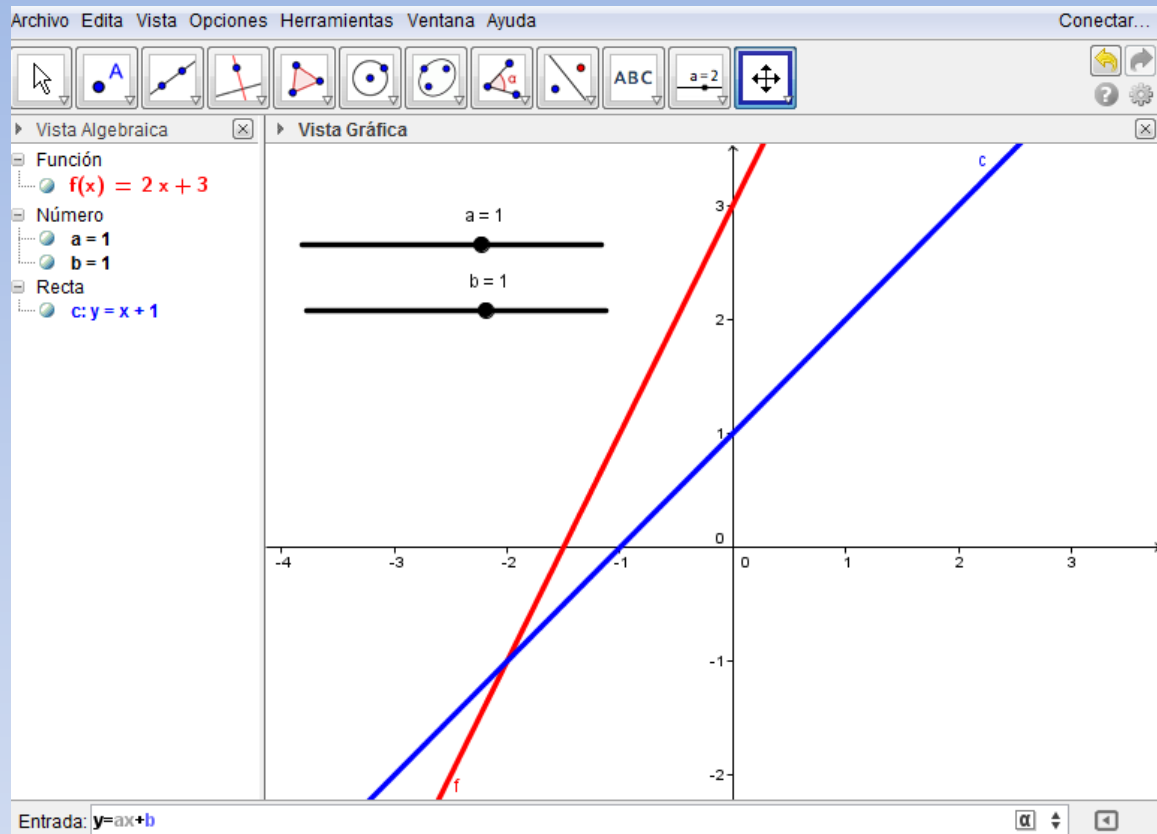
One problem is static because the required task is unambiguous: you have to apply any known technique or method to find the solution (eg, the trite statement "Given the following system of equations, solve ...").



A dynamic problem is constituted as such by the type of relationships that enables to establish the statement of the problem facing the task to which students are invited.

## First problem: A fixed equation and an equation parameterized

Write on the bar into the equation of a line.  
Enter each of the sliders (in the entry bar is placed for example  $a = 1$  and  $b = 1$ , then the ends that may vary each is set) and the equation of a line in function writes the thereof.  
 $y = ax + b.$



Manipulate the sliders to analyze under what conditions different phenomena were obtained: an intersection with the given line, any intersection or that the line overlaps the left.

Enable CAS view. Reply:

What is the difference between entering through the entry bar or writing the equation through the CAS window?

What happens to solve one of the equations? And to solve both? What hides the software resolution?

Are the forms of writing in the input bar and the CAS view the same?

Second problem: A fixed equation and the point of intersection with another function "hidden"

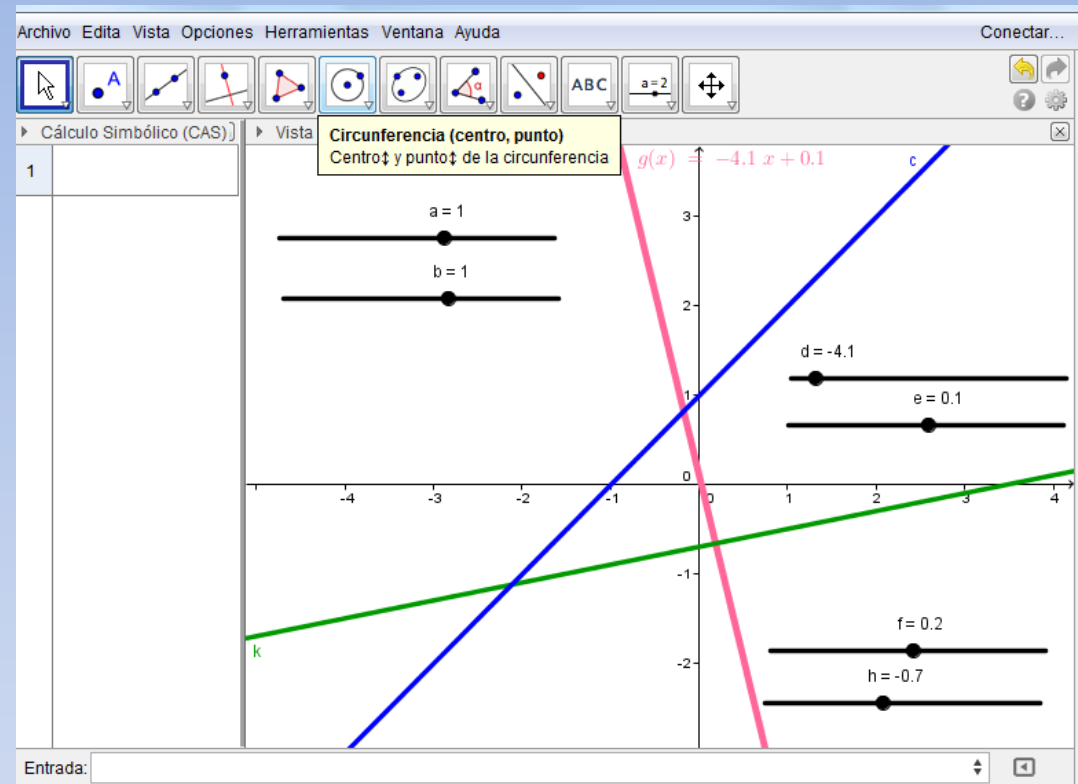
An unusual task is given an equation and intersection, find the other equation.

They are given a file where students from a straight line to a point, which is identified as the intersection with other (linear, quadratic, cubic, etc.) functions is displayed. The given line is to intercept a slider.

The first situation to explore that line is moving through the slider and observe what happens to the intersection point. Is it unique? Why could it be that there is in some cases more than one point of intersection, only one or none? What kind of function intersects the original line? Why?

### Third problem: Three parameterized equations

To begin to broaden the perspective on systems of equations, we propose a situation where there are three intersecting lines. Since all equations are given in parameterized form. The purpose is to analyze the different conditions that must assume the parameters to be varied amount of solutions.



What if two lines are parallel? What if all are parallel? Is it possible that all lines intersect at the same point? What should "see" in the CAS window in the graphical view? Are they compatible?

An alternative approach might be to consider that can leave trace different lines and then "open" the discussion to systems of inequalities and optimization problems.

#### Fourth problem: Incomplete Equations

Given the following system of equations is it possible to find the value of  $a$ ,  $b$  knowing that the solution of the system is the point  $(2, 1)$

$$\begin{cases} 2x + 3y = a \\ 3x + y = b \end{cases}$$

$$\begin{cases} \frac{3}{2}x + 7y = a \\ -2x - \frac{5}{3}y = b \end{cases}$$

Fifth problem: Guessing from a system of equations.

Given the following equation system perform an analysis of the solutions for various integers of  $n$  and  $m$  values.

$$\left\{ \begin{array}{l} Y = X^n \\ Y = X^m \end{array} \right.$$

Workshop with teachers: three sessions of 2 hrs. each. Some thoughts after the first session

At the level of workshop management:

- "Unexpected" Complexity of Instrumental Genesis (Rabardel, 1995) “syntax”
- Artifact - Instrument
- Individual and collective Instrumental Genesis

## **Conclusions:**

The software allows us to extend the range of systems of equations to consider not only linear systems. On the other hand, enables us to make a global approach to the behavior of the functions. At the same time lets us work the issue from other perspectives.

The opportunity to work with the graphical view and the active trace tool, would explore the situation to make a guess and analysis example of how the number of solutions is related to the type of equations involved.

The possibility of working with the CAS view allows us to see many solutions to speculate on the type of equations involved.

This window also allows us to analytically validate what we conjecture graphically.

At the same time holds a potential for dual work than numeric-algebraic.

It remains open discussion between the technical and epistemological value of instrumental techniques, building an outstanding technological discourse.

## Reference:

Artigue, M. (2002). Learning Mathematics in a CAS Environment: the Genesis of a Reflection about Instrumentation and the Dialectics between Technical and Conceptual Work. CAME Forum 2002.

Artigue, M. & al. (1997). Intégration de calculatrices complexes à l'enseignement des mathématiques. Cahier DIDIREM spécial n°3. IREM Paris 7.

Balacheff, N. (1994). La transposition informatique. Note sur un nouveau problème pour la didactique, En: M. Artigue & al. (Eds.), Vingt ans de Didactique des Mathématiques en France, pp. 364- 370. Grenoble: La Pensée Sauvage.

Bifano, F. y Villella, J. (2012). Saberes contruidos con (en) problemas dinámicos: ¿Otros objetos de saber? Colloque hommage à Michèle Artigue. Université Paris Diderot- Paris 7, Francia.

Hohenwarter, M. y Jones, K. (2007). *Ways of linking geometry and algebra: the case of GeoGebra*. Proceedings of the British Society for Research into Learning Mathematics. 27 (3): 16-131, University of Northampton, UK: BSRLM

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: guiding students' command processes through instrumental orchestrations *In: International Journal of Computers for Mathematical Learning* pag. 281-307. Países Bajos, Kluwer Academic Publishers.