



Fifth Central- and Eastern European Conference
on Computer Algebra- and Dynamic Geometry Systems
in Mathematics Education

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THE USE OF COMPUTERS FOR CALCULUS TEACHING

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This work aims at *analyzing the use of computers for teaching Differential and Integral Calculus*.

The analyzed case is a relation between differentiability and continuity in real functions.



Fifth Central- and Eastern European Conference
on Computer Algebra- and Dynamic Geometry Systems
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The example chosen is a non-differentiable continuous function defined in the unit interval.

This example is found in an article written by David Tall (1982).



Fifth Central- and Eastern European Conference
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Through this example is possible to evidence a way in which a computer may help the teaching of concepts of Differential and Integral Calculus through the production of meaningful didactic materials.



Fifth Central- and Eastern European Conference
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Besides that, to build such example we have chosen the software GeoGebra.

This *software* holds all tools and commands which enable the building of the aforementioned function, since it is the limit to a series of functions.



Fifth Central- and Eastern European Conference
on Computer Algebra- and Dynamic Geometry Systems
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26-29 September, 2014 Halle (Saale), Germany

The choice of this example was made because it is contrary to the student's intuition and reinforce that the reciprocal of the following theorem is not valid:

If a function f defined in a real interval is differentiable at a point of this interval, then it is continuous at this point.



Fifth Central- and Eastern European Conference
on Computer Algebra- and Dynamic Geometry Systems
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26-29 September, 2014 Halle (Saale), Germany

The potentiality of the use of computers when teaching advanced Mathematical topics is mentioned by Tall:

[...] to use the computer to visualise mathematical concepts in helpful ways in calculus and analysis. Imaginative use of graph-plotters and graphic calculators has enabled students to cope more meaningfully with concepts such as differentiation through the notion of “local straightness”, integration through area summation, and solving (first order) differential equations by visually building up solution curves with given gradient (TALL, 1993, p. 2).



Fifth Central- and Eastern European Conference
on Computer Algebra- and Dynamic Geometry Systems
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26-29 September, 2014 Halle (Saale), Germany

In Tall's formulations, the graphic representation of the differentiable function, when it enlarged to a determined portion, looks like, locally, a segment of a straight line.

Afterwards, the researcher formulated the notion of "*local straightness*".



Fifth Central- and Eastern European Conference
on Computer Algebra- and Dynamic Geometry Systems
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26-29 September, 2014 Halle (Saale), Germany

According to Tall, this notion would be appropriate to the development of the concept of derivative because “it allows the gradient function to be seen as the changing gradient of the graph itself” (Tall, 2000, p. 11).



Fifth Central- and Eastern European Conference
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By the notion of *local straightness* it would be possible to stimulate the student's imagination to conceive how a graphic representation of a continuous and non-differentiable function at the points of the domain would be.



The definition of the real “blancmange function”

The “blancmange function”, which will be denoted by b , is a function with domain in the closed interval $[0,1]$ and image is in the setting of real numbers, defined by the limit of the series of functions.

$$b(x) = \lim_{n \rightarrow \infty} \sum_{i=1}^n f_i(x)$$

In this case, $f_1: [0,1] \rightarrow \mathbb{R}$, is defined by the following sentence $f_1(x) = |x - \{x\}|$.



The definition of the real “blancmange function”

The “blancmange function”,

$$b(x) = \lim_{n \rightarrow \infty} \sum_{i=1}^n f_i(x)$$

To building of the general term of the sequence of functions $f_n: [0,1] \rightarrow \mathbb{R}$ is given by:

$$f_n(x) = \frac{1}{2^{n-1}} f(2^{n-1} \cdot x)$$



Construction of the “sawtooth function”

In order to build the “blancmange function” it is necessary, at first, to build a real function, which can be called “sawtooth function”.

This function is an example of a continuous function which is non-differentiable at an infinite number of points.



Construction of the “sawtooth function”

In order to build this function by using the GeoGebra *software*, it is necessary use the following pre-definite functions: *round()* and *abs()*.

In the “Input” box, type the following command:

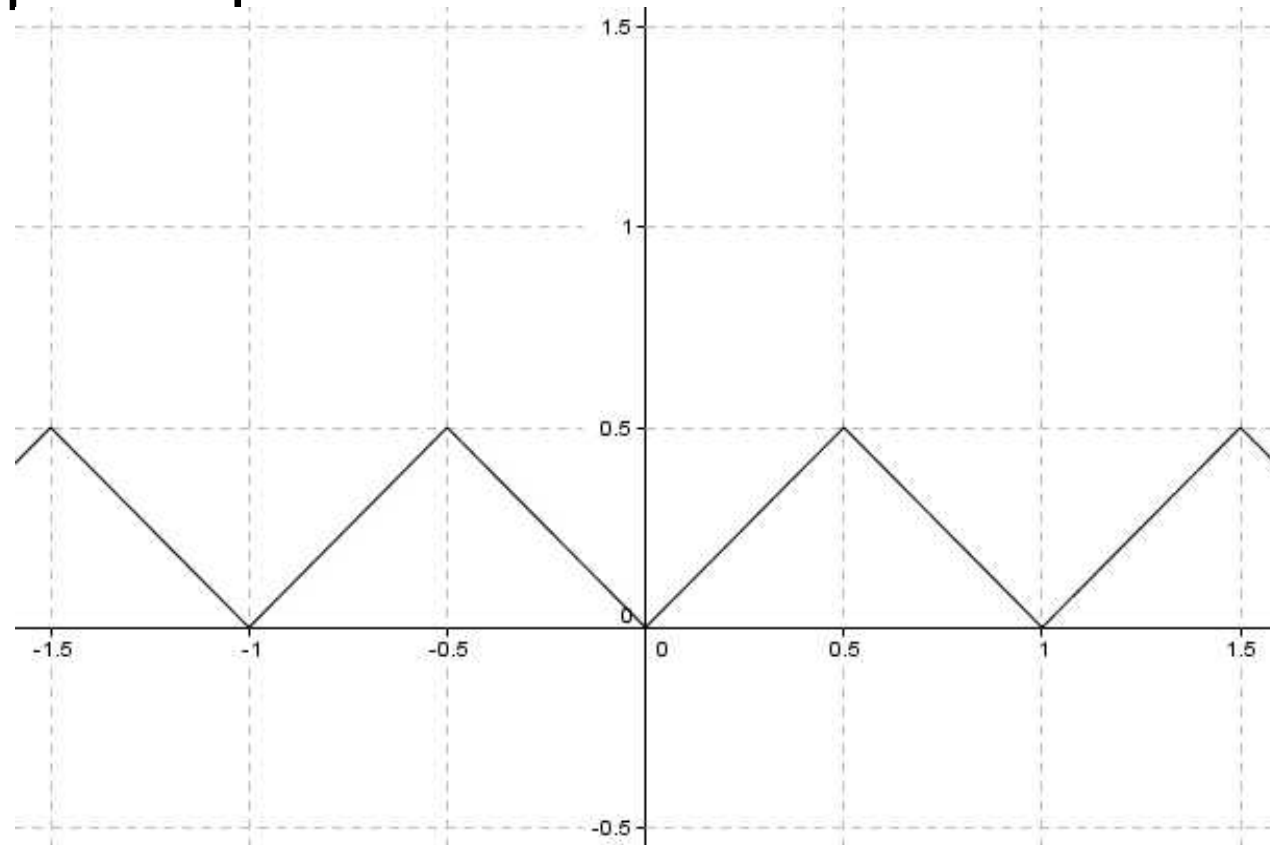
$$“f(x) = abs(x - round(x))”.$$



Fifth Central- and Eastern European Conference
on Computer Algebra- and Dynamic Geometry Systems
in Mathematics Education

26-29 September, 2014 Halle (Saale), Germany

Construction of the “sawtooth function”
The graphic representation of this function is:





Construction of the “sawtooth function”

It is necessary to limit the domain of the f function to the interval $[0,1]$. It is possible to do it in GeoGebra, with the boolean command “If”.

The command brings the following structure: “If[<Condition>, <Then>]”, and it can be used when building a function in which the domain is a subset of real numbers.

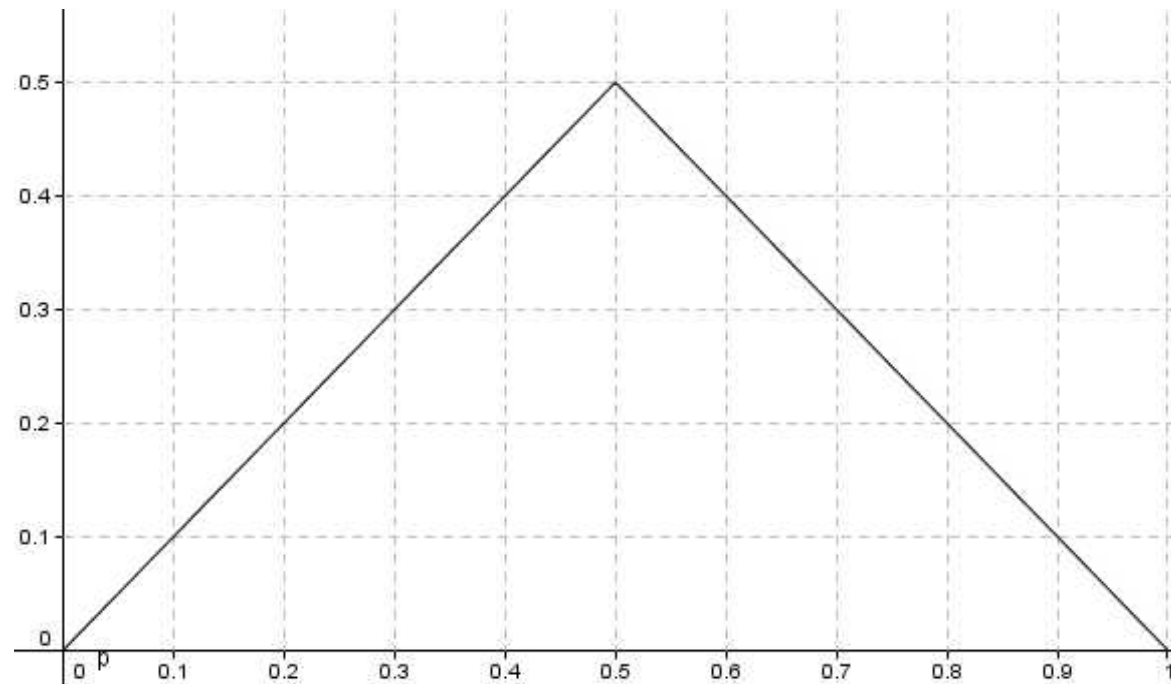


Fifth Central- and Eastern European Conference
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26-29 September, 2014 Halle (Saale), Germany

Construction of the “sawtooth function”

$$f_1(x) = \lfloor [0 \leq x \leq 1, \text{abs}(x - \text{round}(x))] \rfloor$$



The graphic representation of the first term of the sequence of functions, in which the limit is the “blancmange function”.



Construction of the “sawtooth function”

To represent a sequence of functions in GeoGebra,
by means of the command “Sequence”.

“Sequence[<Expression>, <Variable>, <Start Value>,
<End Value>]”



The construction of the “blancmange function”

Example: Consider the sequence of functions $g_i: [0, 1] \rightarrow \mathbb{R}$, defined by $g_i(x) = ix(1-x)^i$ ($i = 1, 2, \dots$).

Type the following commands, in the “Input” box :

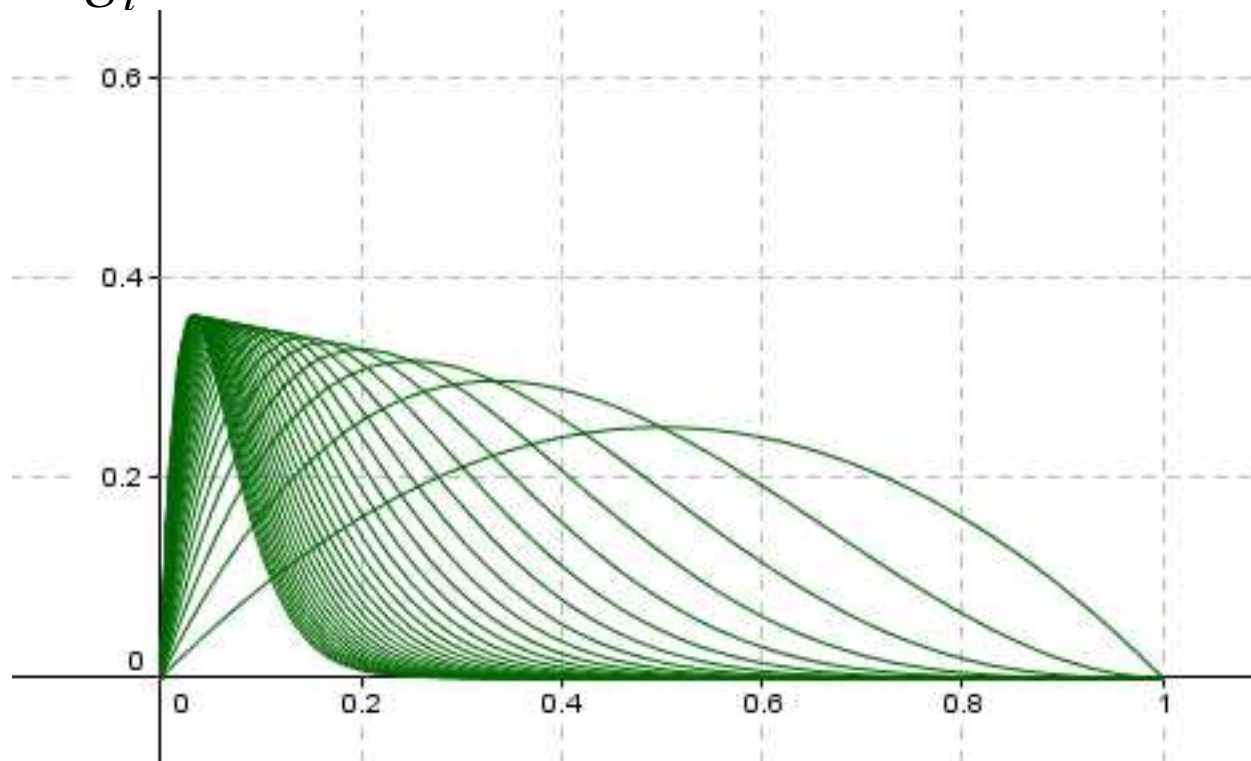
Sequence[If[0 < x < 1, i*x*(1-x)^i], i, 1, 30]



Fifth Central- and Eastern European Conference
on Computer Algebra- and Dynamic Geometry Systems
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26-29 September, 2014 Halle (Saale), Germany

The construction of the “blancmange function”
As a result, the thirty first terms of the sequence of functions g_i .



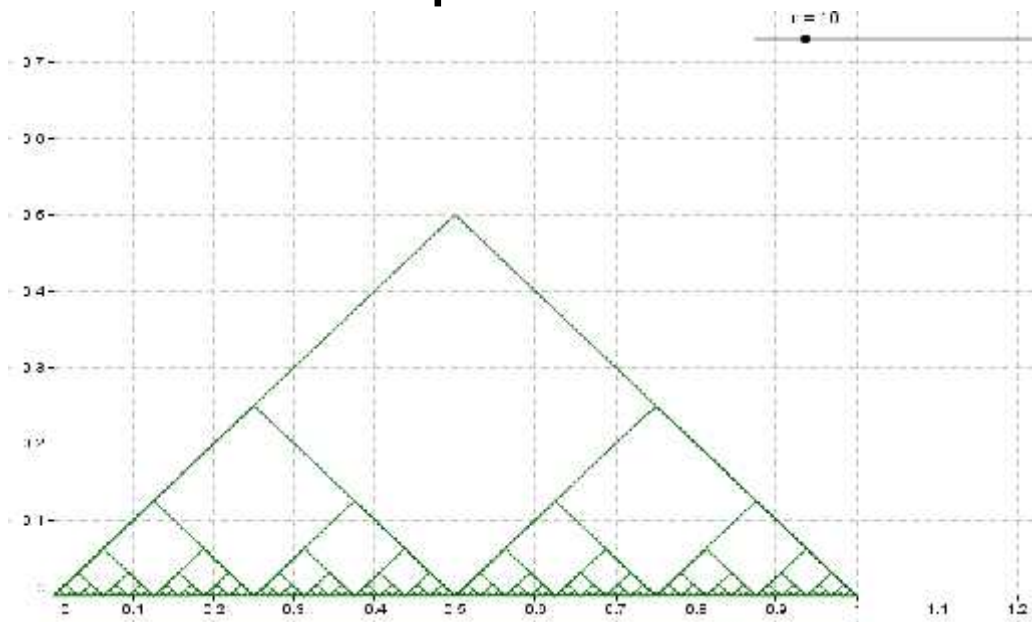


Fifth Central- and Eastern European Conference
on Computer Algebra- and Dynamic Geometry Systems
in Mathematics Education

26-29 September, 2014 Halle (Saale), Germany

The construction of the “blancmange function”

With this command and others details, that will be exposed on final paper, it is possible to construct the 10 first terms of the sequence of functions $(f_n)_{n \in \mathbb{N}}$.





The construction of the “blancmange function”

The built elements is named as *lista1*.

A list, to GeoGebra, is a group of objects created by the user. In this case, the *lista1* is a list with the first n terms of the sequence of functions (Hohenwarter, 2009, p. 58).



The construction of the “blancmange function”

In order to partially sum of the functions f_i you only need to use the command “Sum[<list>]”, because it sums all the elements of a certain list. This way, you only need to type, in the field “Input”, the following:

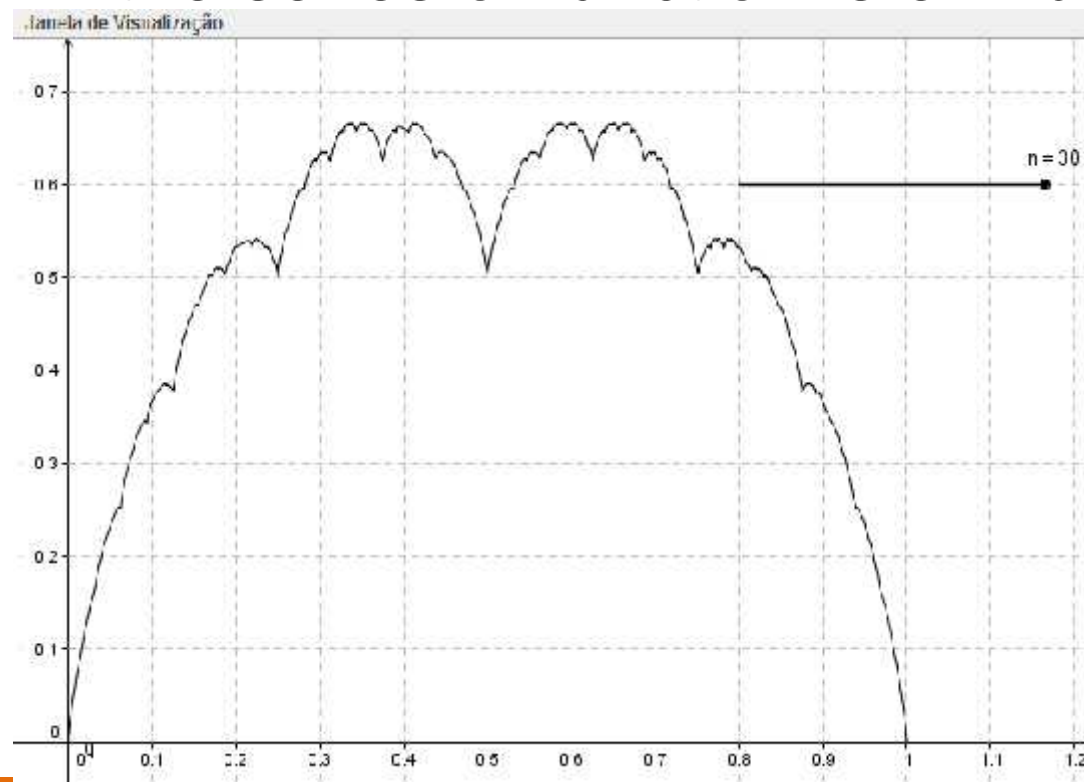
`Sum[lista1]`



Fifth Central- and Eastern European Conference
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The construction of the “blancmange function”
As a result, in the icon “Graphics”, the result of the partial sum in the series of function is exhibited.





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Fifth Central- and Eastern European Conference
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26-29 September, 2014 Halle (Saale), Germany

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