

Lessons Learned In Course »Computer Tools In Mathematics«

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Abstract

Some thought regarding usage of computer tools in teaching and learning mathematics are presented. These thoughts are based on practical experiences with a course entitled "Computer tools in mathematics", which focuses on teaching the students the practical usage of various tools in solving mathematical problems. Some emerged during the preparation of the curriculum for the course. These a questions like, should we use just one tool covering all aspects of teaching and learning mathematics, or many "smaller" tools, how powerful should tools be not to hinder the process of learning mathematics and similar. Other considerations, like establishing proper student's observation of the relation between "mathematical answer" and result obtained from computer tool, the development of support materials and how we can incorporate teaching process in developing resources, emerged when giving the course.

Reader should be warned that this is just a working copy of the paper, far from being finished.

Introduction

In first cycle professional study program Practical Mathematics at Faculty of mathematics and physics, University of Ljubljana, we have a course entitled Computer tools in mathematics. Study program Practical Mathematics is the first cycle professional study program. The primary goal of the program is to produce professionals capable of application of their mathematical knowledge in the working process in communications and information technology, technology, logistics, and elsewhere. There are several subject specific competences to be developed by the student:

- ability of **employment of mathematical tools at practical problem solving**,
- ability of identification of mathematical processes in professional environment,
- ability of **result analysis**,
- ability of **presentation of results**,
- ability of employment of mathematical tools in everyday environment,
- ability of initiation and conduct of software upgrade,
- ability of business process optimization,
- ability of mathematical modelling of technological processes,
- ability of employment of numerical methods at mathematical problem solving

In the preparation of the course, we focused mainly on the highlighted competences. Therefore, the main aim of this course is to show the students the practical usage of various tools in solving mathematical problems. Students will learn about and learned to use computer tools suitable in various mathematical procedures. A quite important aspect is also to learn how to report on their work.

It is in the first year in the first semester with 15 hours of lectures and 30 hours of lab exercises.

Syllabus is primarily focused onto get used to the programs for numerical and symbolic calculations, drawing the graphs of functions, solving mathematical tasks from analysis and algebra with a computer. Tools to be considered are computer algebra systems, spreadsheets, tools for visualization and computer aided dynamic geometry systems. We agreed on the following intended learning outcomes:

- **Knowledge and understanding:** Student is familiar with computer tools and approaches to their usage in mathematical procedures. He is able to report on the process of solving a mathematical problem with computer tools.
- **Application:** The knowledge gained is used to support various subjects.
- **Reflection:** To learn about the importance of computer tools in learning and doing mathematics. To find about the importance of reporting on solving mathematical problems.
- **Transferable skills:** The subject refers to all mathematical subjects and serves for learning tools, useful in the study of these subjects.

There were several decisions we had to make during lectures preparation. Perhaps one of the most important was our aim that students should be not exposed to any new mathematics. Therefore, whenever a certain aspect of a tool has been demonstrated or an exercise given, it was mathematically solvable with student's existing mathematical knowledge. Most examples were taken from high school tests and from already taken coursework of their other mathematical courses.

The last year's content has been as follows:

- Wolfram Mathematica
 - Basic functions (CAS), 2D plots
 - Dynamic applications (Manipulate)
- GeoGebra
 - Basic geometric constructions (triangle and points of interest, n-polygons, Tales theorem ...)
 - Polynomials (zeros, extremes ...)
 - Using Trace (locus, optimization problems)
- Excel
 - Pivot tables
- Numpy
 - Mostly just the idea

A very important part of the course is also their seminar work. The last year they had to:

- actively participate in GeoGebraWiki (<https://wiki.lokar.fmf.uni-lj.si/geogebrawiki>),
- prepare a collection of solutions to mathematical tasks (using computer tools),
- prepare a description of a new tool which can be used in solving mathematical tasks and
- give a live presentation of a new tool (10 minutes)

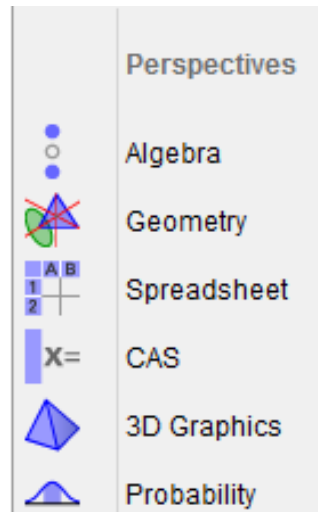
Curriculum design questions

In this part we present some observations we have during the preparation of a course as well as during its revision which takes place each year after the school year ends. The main question was – one or many tools. The basic problem we had during the design of the course has been a decision

should we choose one powerful tool and equip the students with detailed knowledge of this particular tool or should we use many different tools, each one for a certain topic.

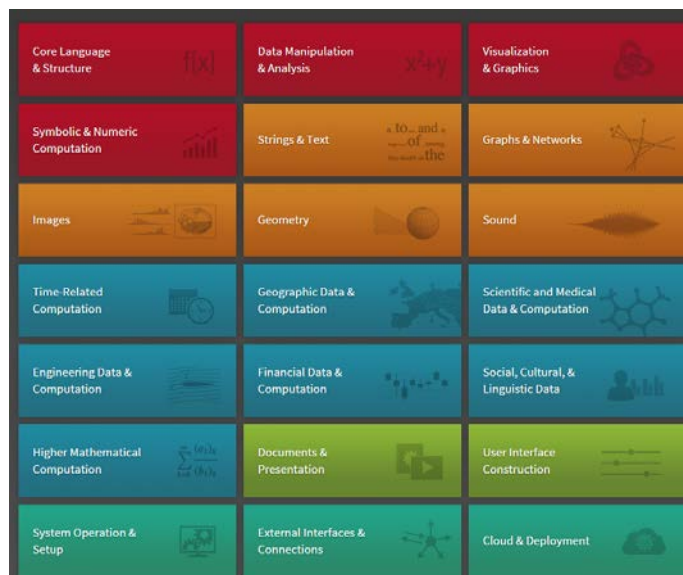
One tool covering all aspects of teaching and learning mathematics, or many "smaller" tools

Namely it seems that with time all software intended to be used as a math teaching tool acquires more and more features. For example when GeoGebra is started it offers different perspectives



PICTURE: 1 POSSIBILITIES IN GEOGEBRA 5

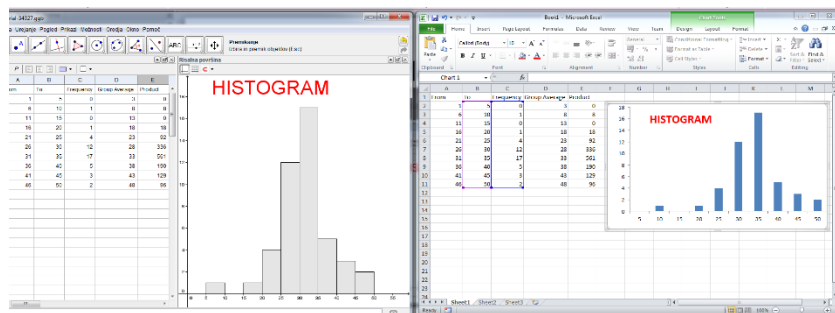
But there are also tools covering even broader spectrum of possible work, Wolfram Mathematica for example



PICTURE: 2 - MATHEMATICA 10

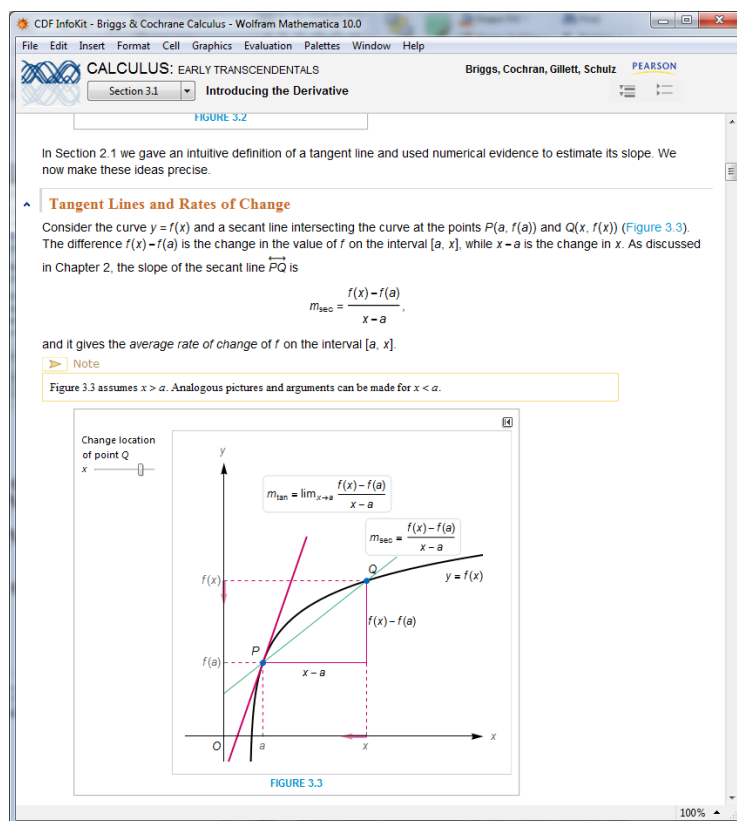
Some teachers claim it is best to stay in the same environment all the time. However, is that really the case? Is not part of the teaching process equipping students with the knowledge of how to pick the right tools for a particular task?

Is it really more convenient to learn how to produce a histogram in GeoGebra than using probably simpler, but anyway more powerful tool for same task in Excel (or Libre Office Calc or similar)?



PICTURE: 3 USING GEOGEBRA OR EXCEL

I am also quite impressed with Mathematica's capabilities to produce professional looking papers equipped with dynamic examples made by Manipulate command. But it requires also quite considerable learning effort to polish paper to such extent. Quite often much simpler (regarding the time invested in learning the tools) combination of "ordinary" word processor with a link to CDF file

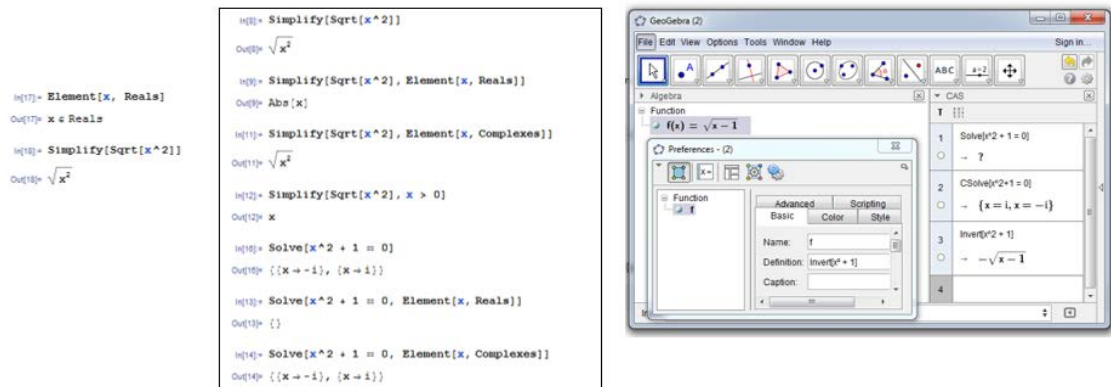


PICTURE: 4- RICH MATHEMATICAL TEXT IN MATHEMATICA

And aren't smaller tools often more flexible and versatile? However, how does one then switch the result between the tools? For example – how to export the data obtained in GeoGebra to Excel to produce the diagram.

How powerful should tools be when they are considered teaching tools?

When a certain tool is meant as a teaching tool – should its capability be limited to the level of math knowledge of the students? On the other hand, should the students be exposed to "real life" tools without limitations? What about the possibility of posing limitations to and "fine tuning" the tools' capability?



Interoperability of tools

When we would like to solve a certain task where at each step a different tool is used – how do we switch the results between the tools? Is it possible to clearly define a math object's specification that would support such interoperability?

One of the most fruitful discussion with the students were the hours when we considered how to solve the following task:

In GeoGebra points A, B, C and D are chosen. Construct a cubic polynomial through those points. Draw a tangent line in arbitrary point $(x, p(x))$.

Help: coordinates – use $x()$ and $y()$. To calculate the polynomial use MATLAB.

Some observations of the student's work design questions

Relation between “mathematical answer” and result obtained from computer tool – some practical examples how students internalize the notion of “correct result” when working in different environments with different tools will be given.

Exactness of the solutions is often problematic when we use tools. Students have significant problems differentiating between exact and approximate solutions, especially when approximate solution is obtained using a computer tool – something we have not observed so often when we used “non computer tools” (rulers, compass, logarithmic tables ...).

Zbirka Nalog - ROM-SN

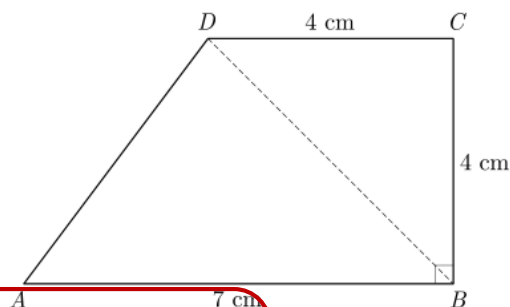
Maturitetna pola zima 2009 - PR- naloga3 (2.del) - Geogebra

Podatki o nalogi:

- Poklicna matura
- Maturitetna pola 11.februar 2010, poklicna raven
- 3.naloga (2.del)
- Snov: Pravokotni trikotnik

Navodilo

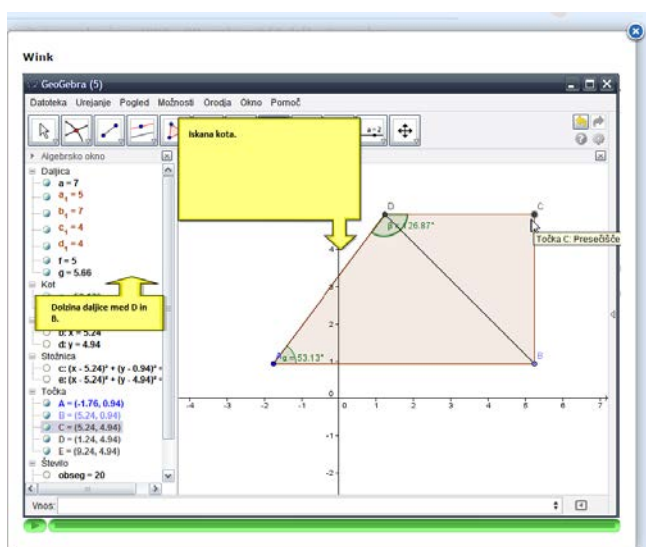
Na skici je trapez ABCD s podatki:



- Izračunajte obseg in ploščino trapeza.
- Izračunajte notranja kota trapeza v ogliščih A in D.
- Izračunajte natančno dolžino diagonale BD.

PICTURE: 5 TAST TO BE SOLVED WITH GEOGEBRA

Student used GeoGebra for solving the task depicted above. The task required to calculate the exact inside angles as well as the length of the diagonal BD. It took quite substantial effort from the lecturer to explain why the solution (given in Picture: 6 "Solution") where he simply made the construction and measured the angles and length is not the appropriate one.



PICTURE: 6 "SOLUTION"

When students has been asked to solve same task with a compass, ruler and a protractor, he immediately argued »it cannot be done as the task requires the exact answer«

Another problem is also the notion of a proof. Quite often students using CAGD tools are convinced that if a construction with a set of chosen points supports the claim, it is immediately the proof of a claim.

Take the following task for example. An arbitrary triangle with certain properties is described. It is required to calculate the angle BFE.

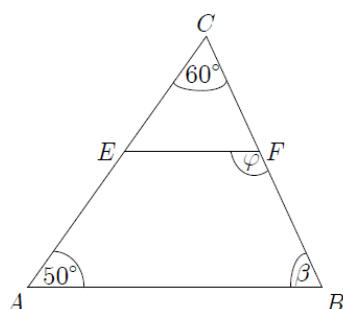
Zbirka nalog



Naloga 2 - GeoGebra - Uvod - Teorija

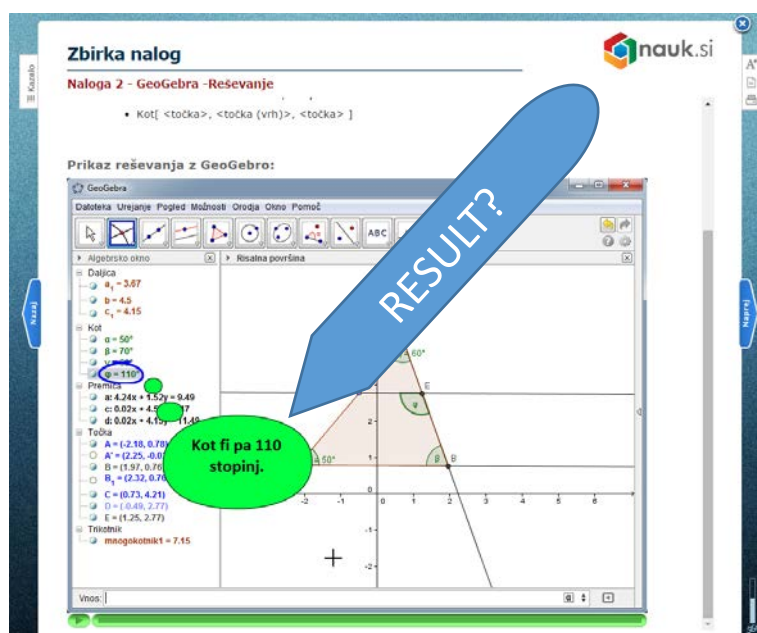
Besedilo naloge:

Dan je trikotnik ABC . Daljica EF je vzporedna stranici AB . Izračunajte velikosti neznanih kotov β in φ .



PICTURE: 7 TASK WITH A CONSTRUCTION

Significant number of students just do the construction and without hesetation simply measure the angle, not even trying to move points A, B, C to see if the angle "stays the same".



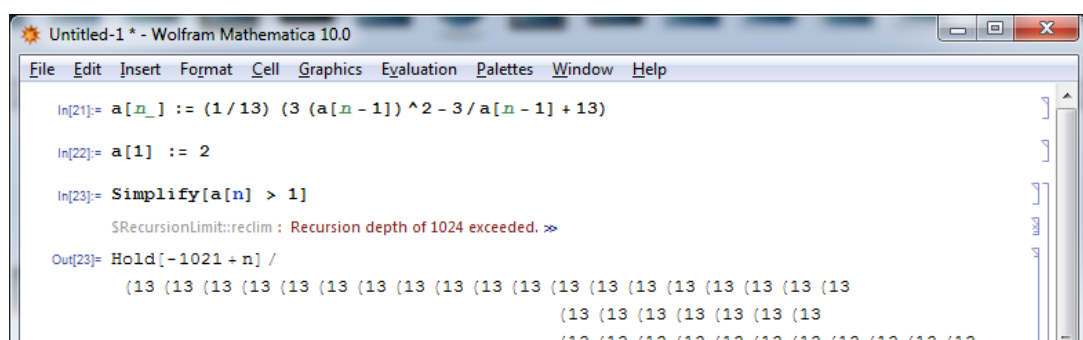
PICTURE: 8 - ONE CASE - AN IMMEDIATE SOLUTION?

presented to a task requiring to confirm that the all terms of the sequence $a_n = \frac{1}{13}(3a_{n-1}^2 - \frac{3}{a_{n-1}} + 13)$, $a_1 = 2$ are between 1 and 3.



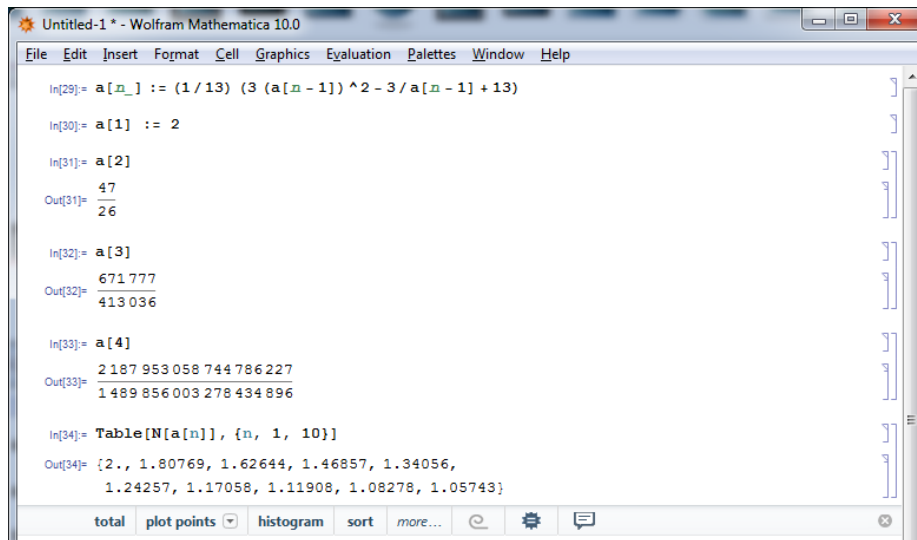
PICTURE: 9 WRONG USAGE OF A TOOL

The student did not realized that Mathematica »confirmed« due to hiss erroneous definition of the terms! Another student, solving the same task, has been luckier, as he got



PICTURE: 10 - MATHEMATICA IS USELESS

But his conclusion “Mathematica is useless for such tasks” has been wrong. It was a complete surprise for her when the lecturer showed how Mathematica can tabulate the terms



PICTURE: 11 - TABULATION OF THE TERMS

Another big problem for students is properly expressing the results. Using computer tools quite often leads to even worse situation due to student's inability to properly interpret the output of a tools. A simple example is a task requiring solving

$$(x + 2)/(x^2 - 2x + 1) = 2/(x - 1)$$

Student presented the solution in a form "Equation holds for $x = 4$ and does not hold for $x = -1$ ". When she has been asked why did she use such wording, she explained "Quick math wrote $x \neq 1$ beside the equation and at the bottom of the steps $x = 4$ ".

Support materials development and usage

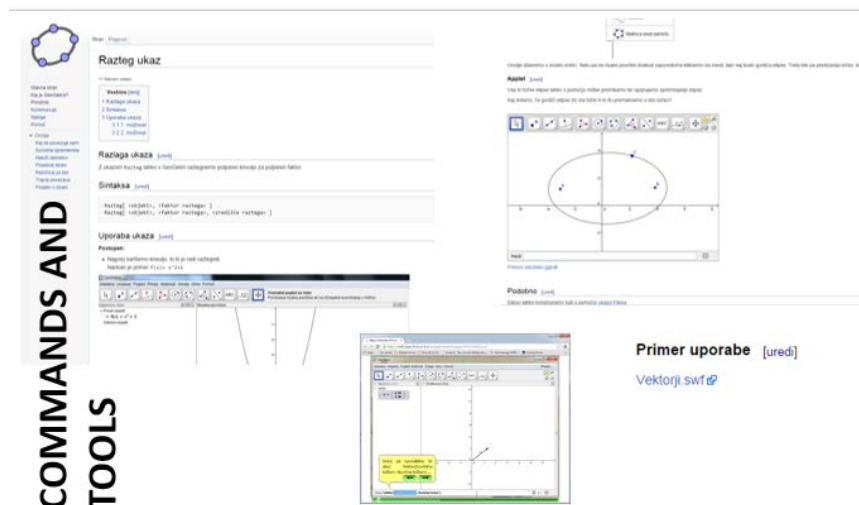
What are the best ways of developing support materials? How we can incorporate teaching process in developing resources. We were quite surprised how the level of students' capabilities to use has risen after they were required to participate in developing GeoGebra Wiki (<http://lokar.fmf.uni-lj.si/wiki/GeoGebraWiki>). This is a wiki in Slovene language where explanation of GeoGebra commands and tools with examples is given, different hints and various interesting constructions (with detailed explanation how we make it) presented. There is also a collection of GeoGebra aided solutions to various tasks mostly from matura external examination.

The following pictures briefly describe our wiki.

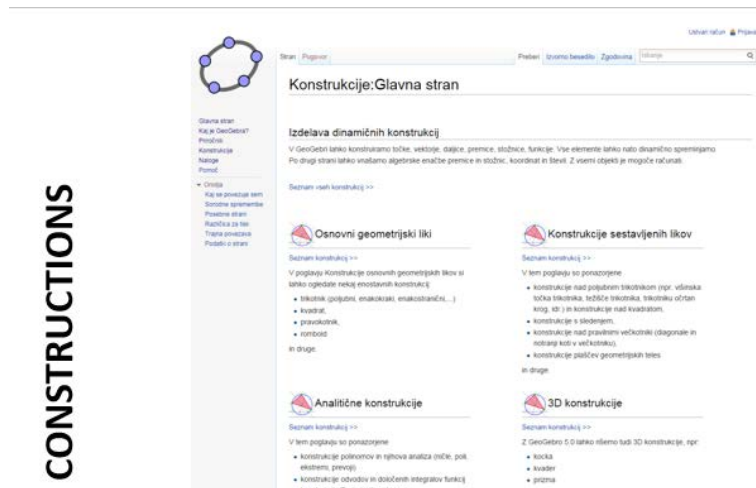
<https://wiki.lokar.fmf.uni-lj.si/geogebrawiki>



PICTURE: 12 MAIN LAYOUT OF OUR GEOGEBRAWIKI



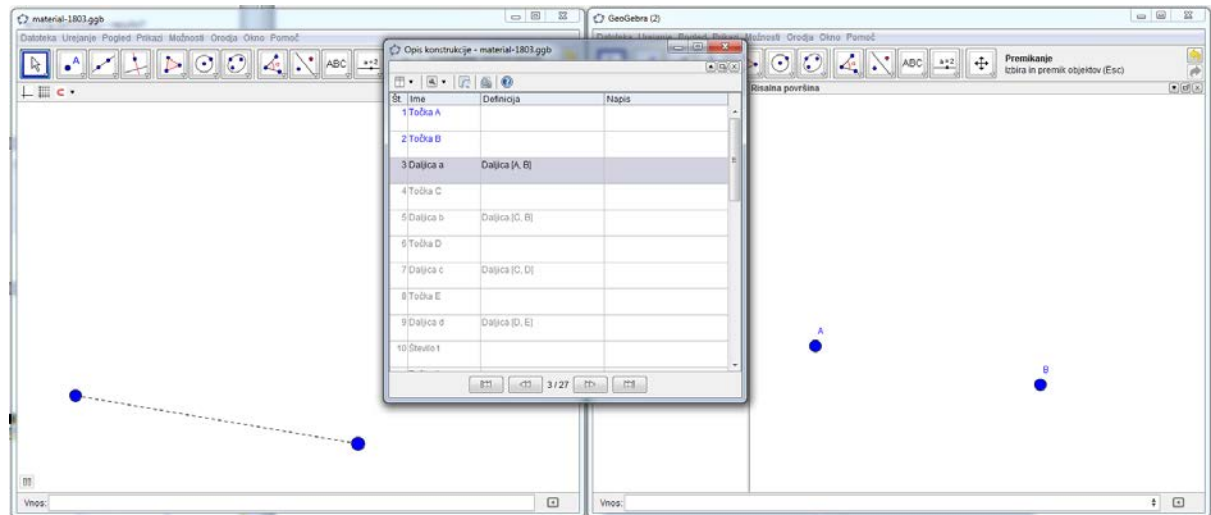
PICTURE: 13 TOOLS EXPLAINED WITH EXAMPLES, SCREENCASTS, HINTS...



PICTURE: 14 - DESCRIPTION OF VARIOUS CONSTRUCTIONS

Openness of the construction process

One of the task for students is also to prepare detailed explanation of approach in solving a certain mathematical problem, we observed several problems students have. Besides the obvious ones like the difficulties in using appropriate mathematical language, there are also “technical” ones. For example: is the way GeoGebra offers the possibility to see the construction steps the best possible one? What about the “intermediate steps” such as hiding certain objects or changing the properties which are not recorded ...



PICTURE: 15 LACK OF CERTAIN IMPORTANT STEPS (PROPERTIES OF OBJECTS...)

Conclusion

This has been just a brief walkthrough of some observations we made during preparation and lecturing our course Computer tools in mathematics.