Computer Algebra and Dynamic Geometry in Mathematics Education,
Halle, Sept. 27, 2014

'Dynamic geometry and mathematics: few trains on a two-way track'

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Preliminar

CADG =======> ME

Back to Spain

ASSOCIATION FOR PUBLIC POLICY ANALYSIS AND MANAGEMENT (APPAM)
Segovia, Sept. 29-30, 2014

The Decline of the Middle Classes
Around the World?

> Informing an Educational Equity Agenda: The Instructional Pipeline from Schools to Teachers to Students

H.H. (MQI)

NYTimes, May 27, 2014

PIAAC

Summary

Dynamic Geometry => Mathematics

• Dynamic Geometry ==> Mathematics ?

What is it used for in the **math** classroom?

mathematics = ?

• Arithmetic operations with numbers, fractions, radicals, factorizing, gcd...
• Get limits, derivatives, integrals, graph funcionts...
• Compute probabilities, averages, deviations..
• Find equations of geometric objects verifying given conditions...
• Solve equations and system of equations, compute determinants, ranks...

If mathematics = operations, computing…

Math and computation, what is the difference?

Día Escolar Matemáticas 2014

```
> evalf(Pi,);
```

• We need to make them think, first, about the difference..

show

visualize obras de arte objetos matematicos

Truncated cube

inquiry

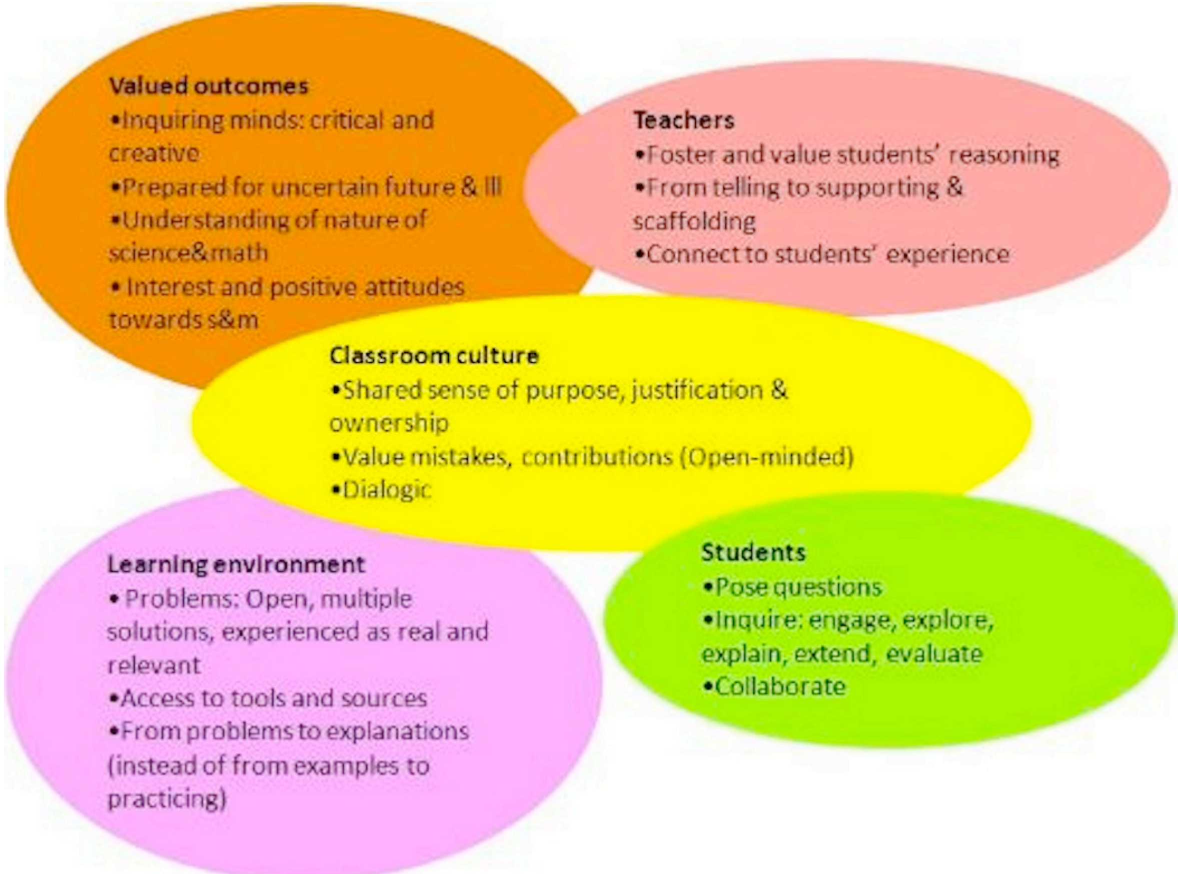
IBL

FP7 SiS

- FP7 Science in Society (SiS)

Report Rocard (2007): "The science education community mostly agrees that pedagogical practices based on inquiry-based methods are more effective, the reality of classroom practice is that in the majority of European countries, these methods are simply not being implemented."

=> European Commission "will support actions to promote the more widespread use of problem and inquiry-based science teaching techniques in primary and/or secondary schools as well as actions to bridge the gap between the science education research community, science teachers and local actors in order to facilitate the uptake of inquiry-based science teaching." (WORK PROGRAMME 2009 Science in Society, SiS-2009-2.2.3.1)



[Artigue, Baptist ,]()

[Artigue, Blomhoj]()

▶ *PRIMAS, FIBONACCI, CT4M*

▶ ***some activities***

▼ ***reason***

▶ *Inscribed triangle*

▶ *Locus Watt*

The cube

Arranz, J. M., Losada, R., Mora, J. A., Recio, T., & Sada, M. (2011). "Modeling the cube using GeoGebra."
En: L. Bu & R. Schoen (Eds.), Model-Centered Learning: Pathways to mathematical understanding using GeoGebra (pp. 119-131). Rotterdam: Sense Publishers.

MODEL at http://www.geogebratube.org/student/m32421

Classic

The flexible cube

Bis

Compare

Some geometric issues

Interesting because:

-elementary
-everyday object
-case of infinite solutions for polynomial system
-intuitive interaction algebra / geometry (and viceversa)
-difficulties with GeoGebra (and with CAS!)

> Pending: assigning one dof to every semi-free vertex.

"Gauss" remarks...

Remarks

- Anecdotical?

[Mathematics ==>] **CADG ==> Mathematics**

"Some Introductory Remarks on Computer Algebra" Wolfram Decker,

"On Lovelace, Babbage and the origins of computer algebra", in Computer Algebra Systems, A practical guide. Edited by Michael Wester, J. Wiley. 1999. pp. 323-331)

Industrial revolution

How round is...

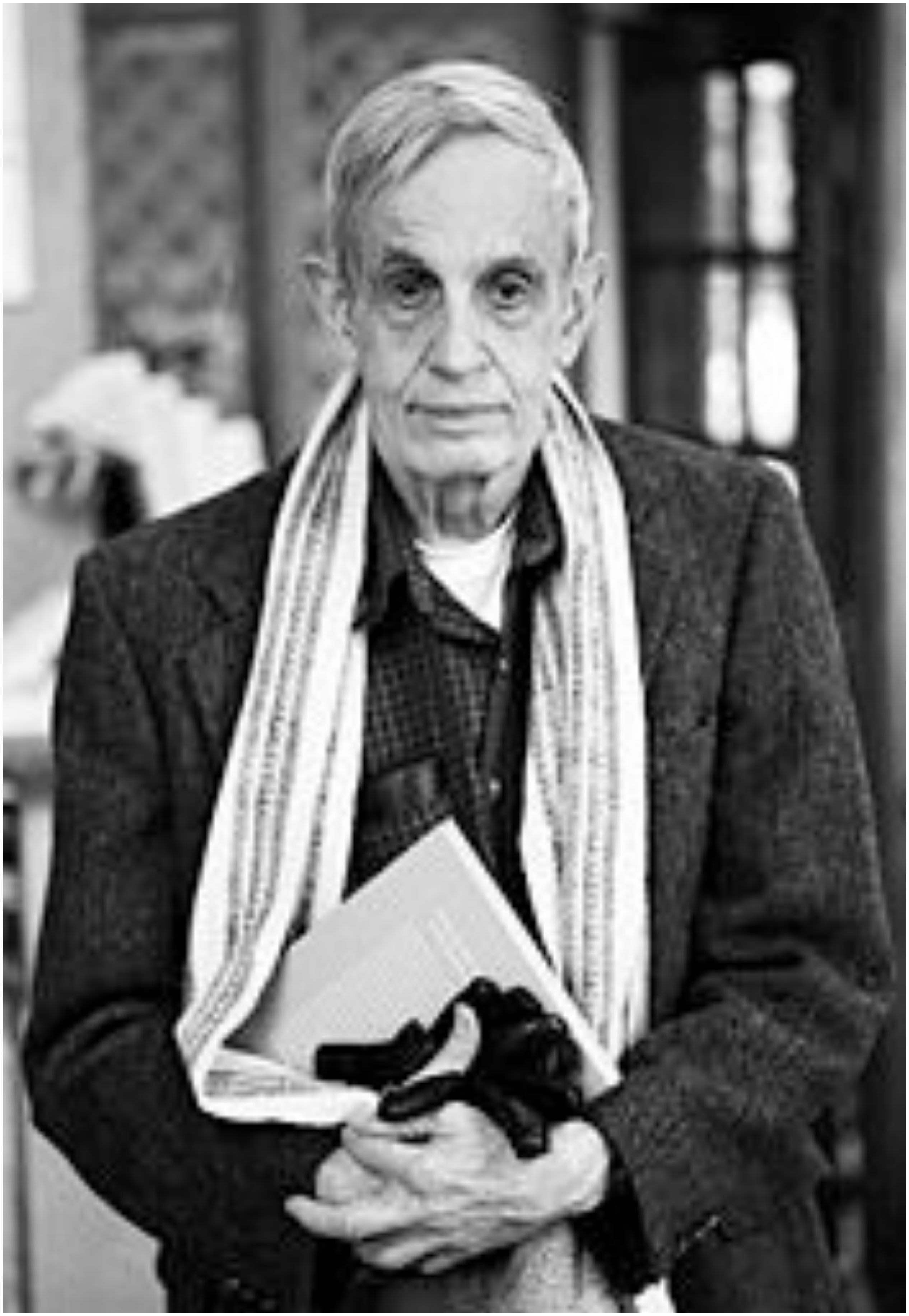
Universality

Conjecture of Thurston (Universality of mechanisms):

“Let M be a smooth compact manifold. Then there is a linkage L whose moduli space is diffeomorphic to a disjoint union of a number of copies of M”. (Kapovich-Millson, 2002)

Otro....

A beautiful mind



http://en.wikipedia.org/wiki/John_Forbes_Nash,_Jr.
http://www.math.princeton.edu/jfnj/

Game Theory, 1994 Laureate Nobel Prize Economics

Nash also did ground-breaking work in the area of *real algebraic geometry*:

"Real algebraic manifolds", Annals of Mathematics 56 (1952), 405–421. MR0050928
See also Proc. Internat. Congr. Math. (AMS, 1952, pp 516–517).

• Nash-Tognoli
Let M be a compact smooth manifold. M is diffeomorphic to a real algebraic set.

• *Nash functions*

$$f(x): R^n \text{--------}> R$$

analytic and $P(x, f(x))=0$, *for some polynomial* $P(x,t)$

▶ *Nash functions*

Quantifier Elimination, projection of real algebraic sets

▼ Mathematics => Dynamic Geometry

Goal

Provide DG with features such as:

- Check if some given statement is true or false

H ==> T?

- Obtain all conclusions from a geometric diagram (or picture)

Given H, find all T's such that H ==> T

- Find complementary hypothesis for the truth of a conjectured statement

Given H and T, find H' such that H & H' => T

--geometric locus computation (enveloppes)

EPO

OMNISCIENT.... as a teacher!

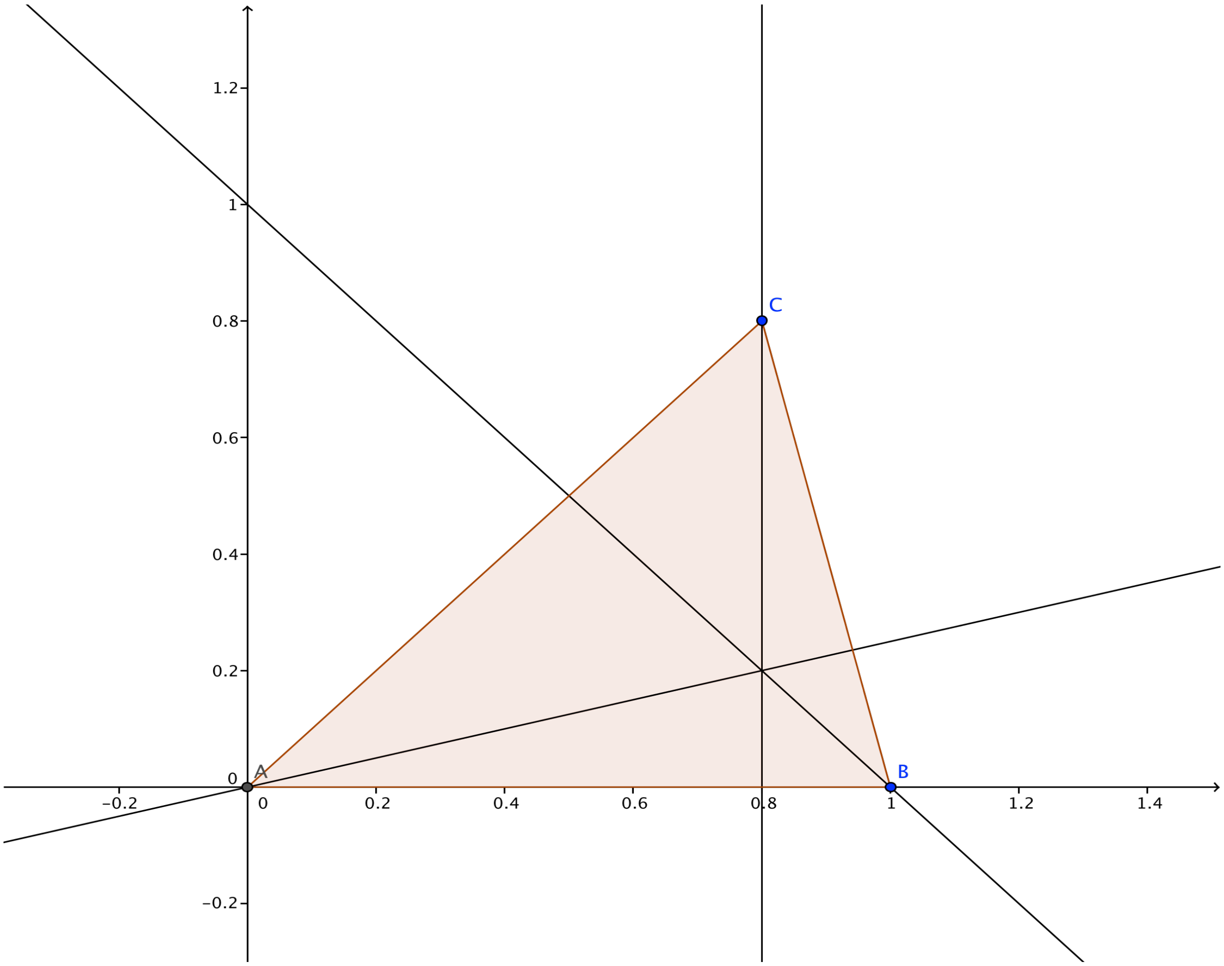
Precursors

Method (idea)

Polynomials

Instances

Triangle A(0,0), B(1,0), C(c1,c2)
Height from A: (c1-1)*x+c2*y=0
Height from B: c1*(x-1)+c2*y=0
Height from C: (x-c1)=0



Intersection of the three heights: there is a solution.

```
> with(linalg): transpose(matrix(2,3,[c1-1,
  c1,1,c2,c2,0]));rank(%);
```

$$\begin{bmatrix} c1-1 & c2 \\ c1 & c2 \\ 1 & 0 \end{bmatrix}$$

$$2 \tag{4.3.2.1}$$

```
> matrix(3,3,[c1-1,c2,0,c1,c2,c1,1,0,c1]);
  rank(matrix(3,3,[c1-1,c2,0,c1,c2,c1,1,0,
  c1]));
```

$$\begin{bmatrix} c1-1 & c2 & 0 \\ c1 & c2 & c1 \\ 1 & 0 & c1 \end{bmatrix}$$

$$2 \tag{4.3.2.2}$$

Thus, for each value of (c1,c2) --except degenerate cases-- there is a solution in (x,y).

```
> EliminationIdeal(<(c1-1)*x+c2*y,c1*(x-1)+
  c2*y,(x-c1) >,{c1,c2});
```

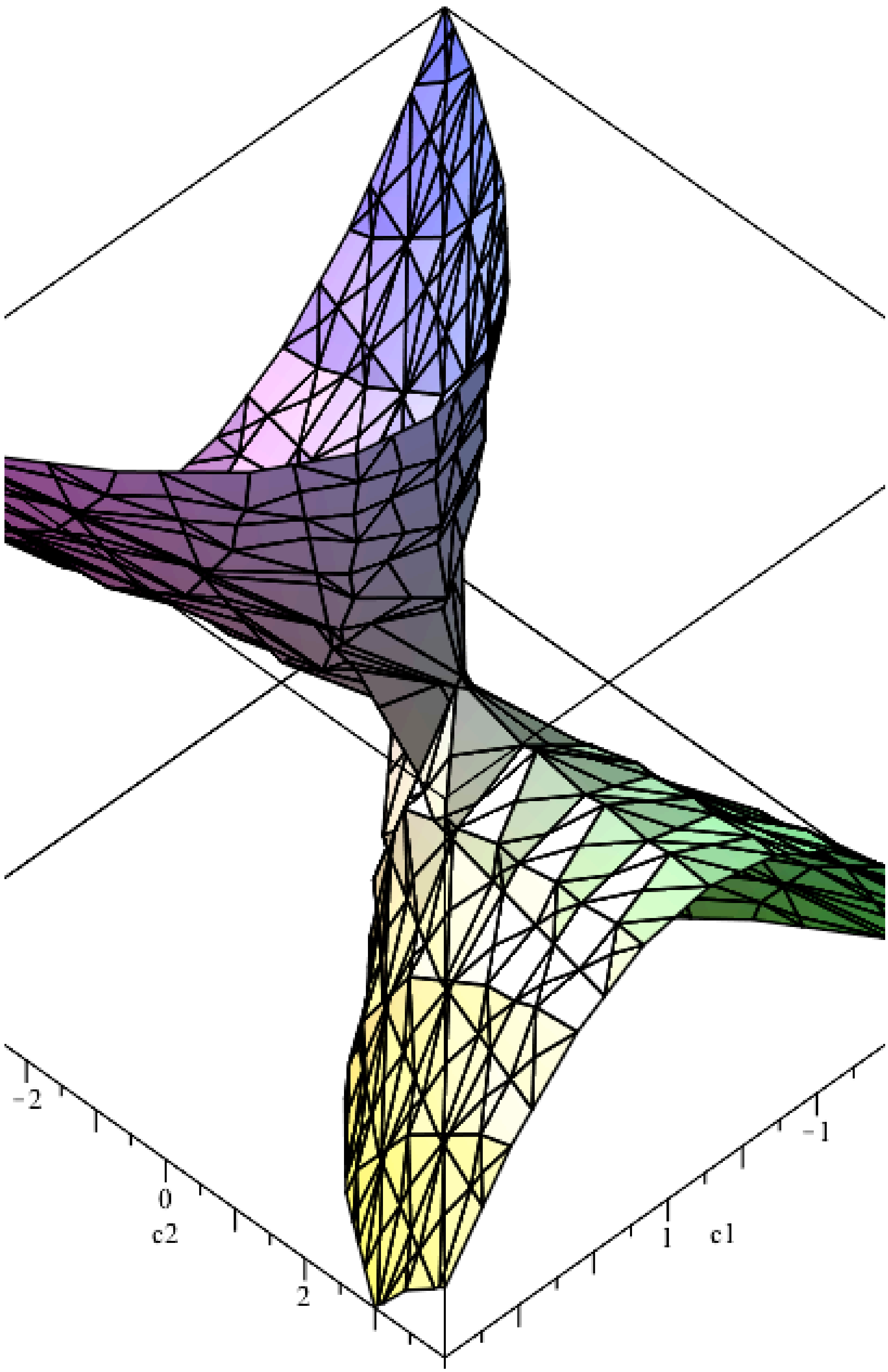
$$\langle 0 \rangle \qquad \textbf{(4.3.2.3)}$$

Now assume we do not know the equations and we do not want to find them, we only know their degree in {c1, c2, x, y} is less or equal than 2.

```
> subs(x=c1, {(c1-1)*x+c2*y,c1*(x-1)+c2*y})
  ;
```

$$\{c1\ (c1-1) + c2\ y\} \qquad \textbf{(4.3.2.4)}$$

```
> with(plots): implicitplot3d(c1*(c1-1)+c2*
  y,c1=-4..4,c2=-4..4,y=-4..4);
```



-2
0
c2
2
-1
1
c1

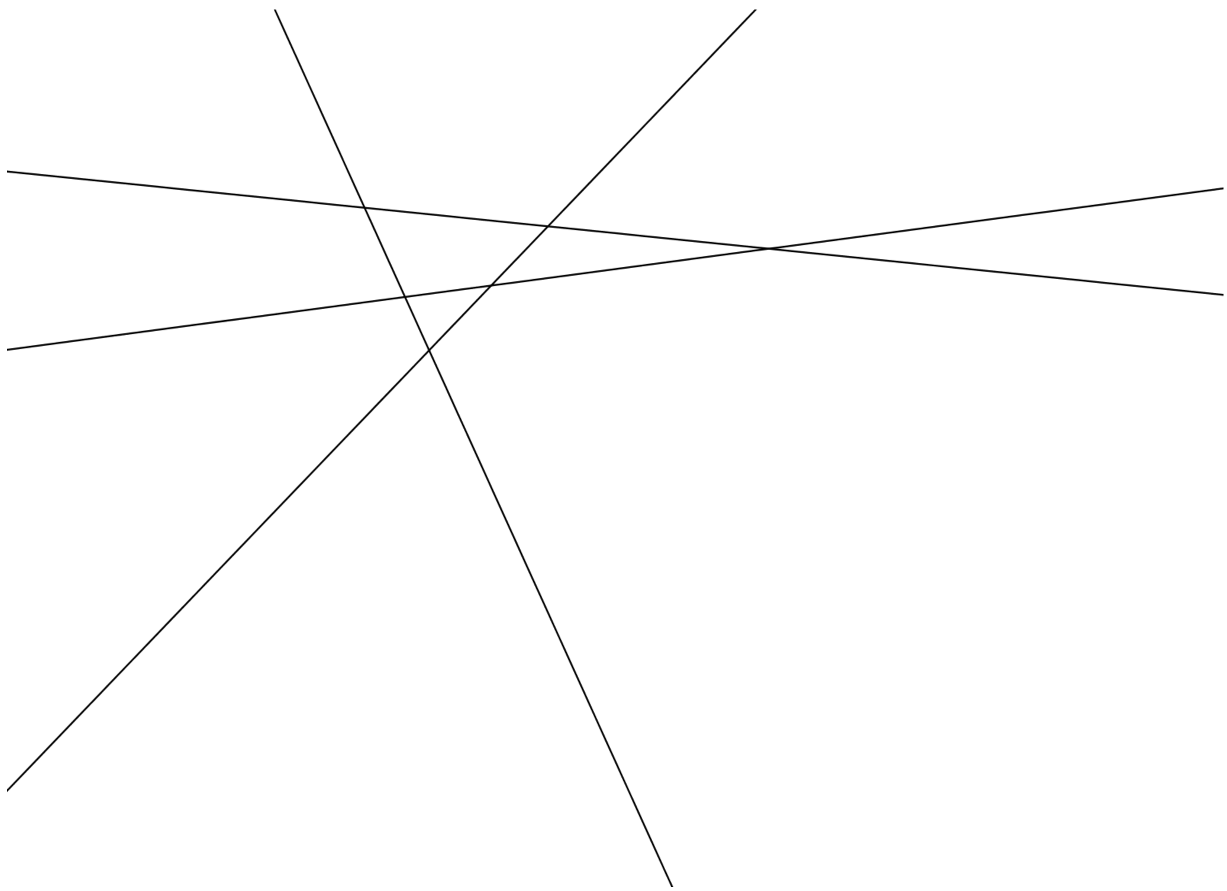
Thus, its projection in the {c1,c2}plane, IF NOT THE WHOLE PLANE must be a curve of degree at most two. Thus it is enough to check the construction for

$$\binom{2+2}{2}$$

six points of integer coordinates, (c1, c2), not on a conic:

-- if the statement is true for these points, then the supposed conic passes through these points, thus it does not exist, and the statemet is generally true and the heights meet at one point.

--if it is not zero at some of these instances, the statement is false or it is a degenerate case.



http://ggb1.idm.jku.at/%7Ekovzol/data/provedetails-20130910/

http://ggb1.idm.jku.at/~kovzol/data/Prove-20140710/
http://ggb1.idm.jku.at/~kovzol/data/prove-provedetails-20140120/README

The importance of being zero

F(x) one variable polynomial,

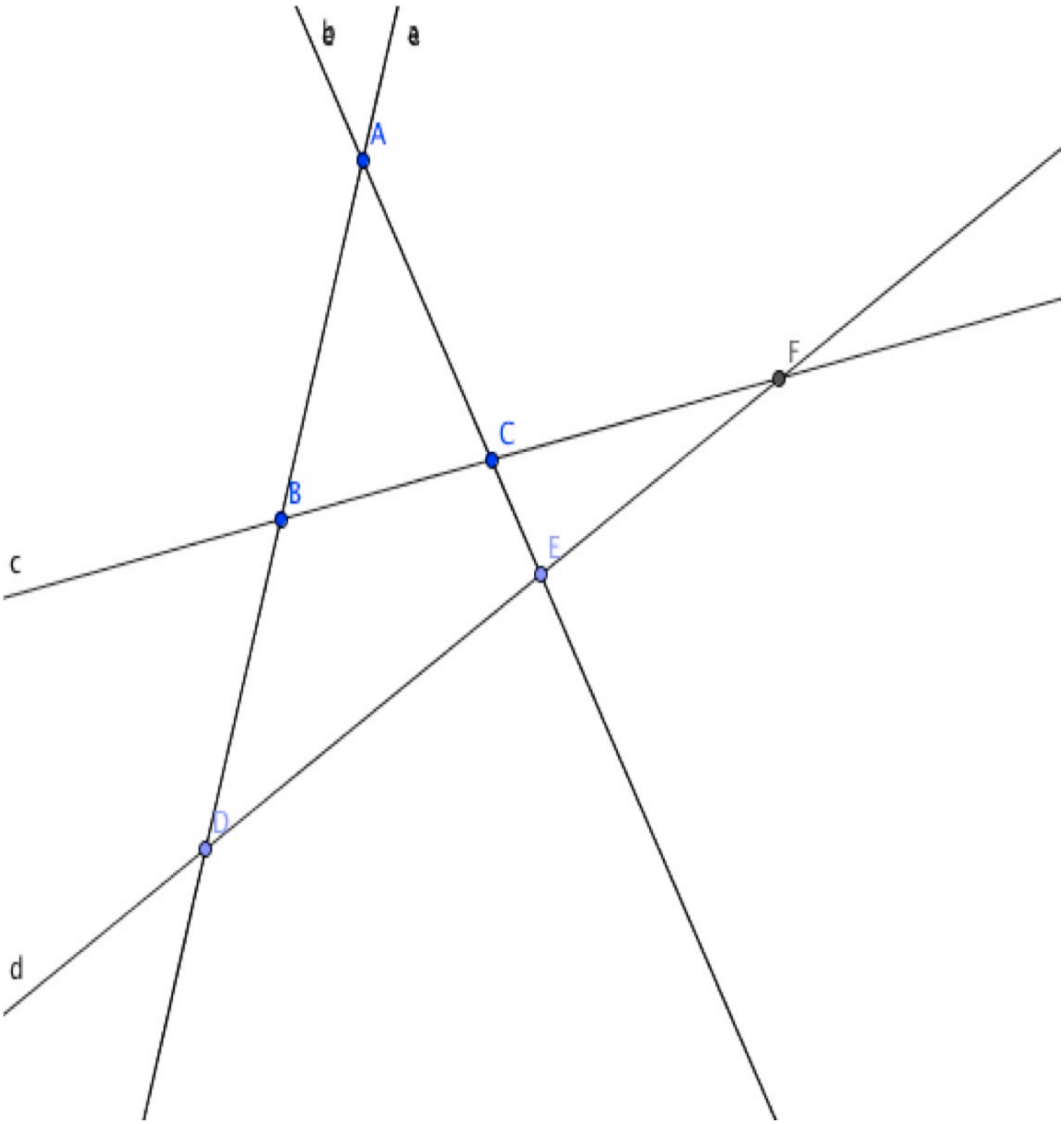
If

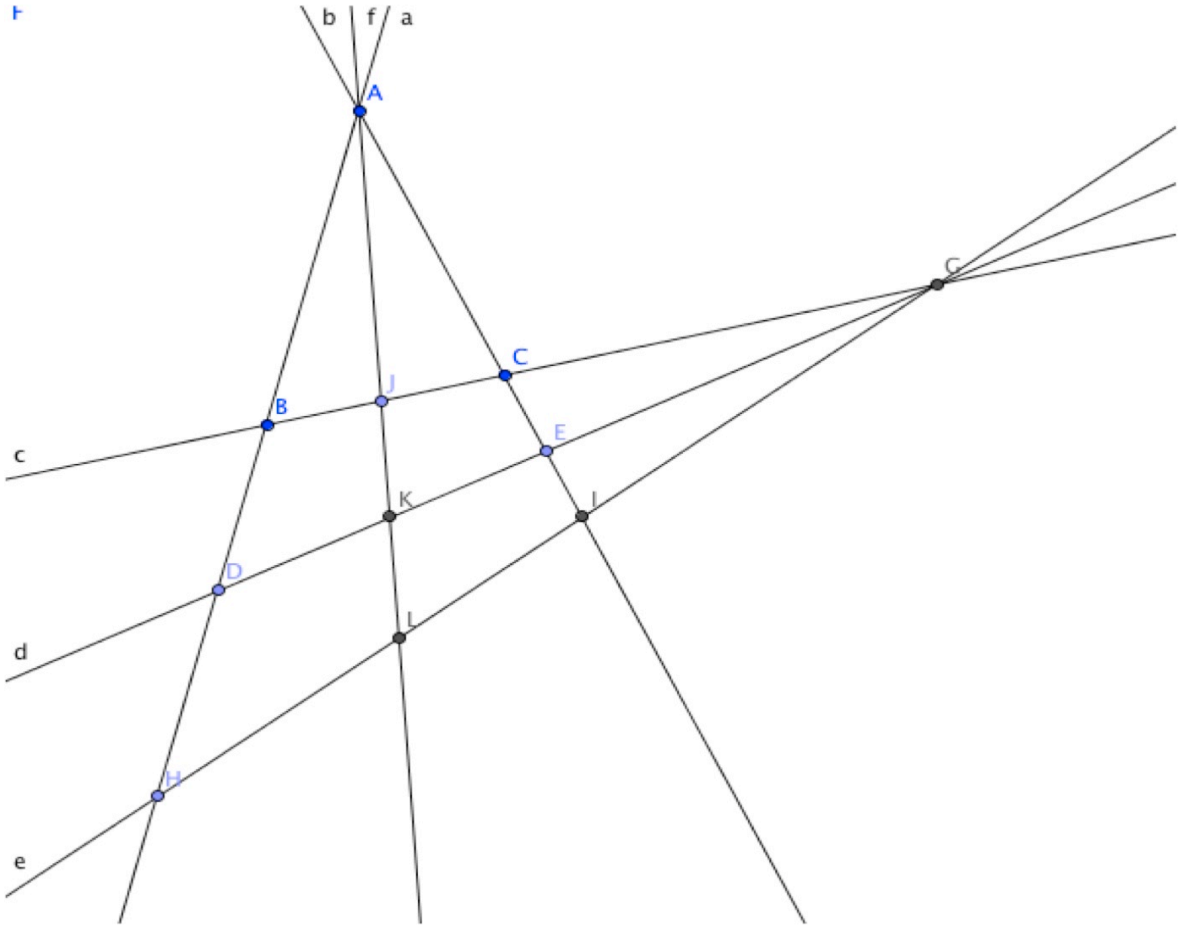
$$\deg(F(x)) \leq d \quad \text{or} \quad F(x) \text{ is zero,}$$

and

$$F(x) \text{ has } d+1 \text{ roots,}$$

then F(x) is identically zero.


b
a
A
F
C
B
c
E
D
d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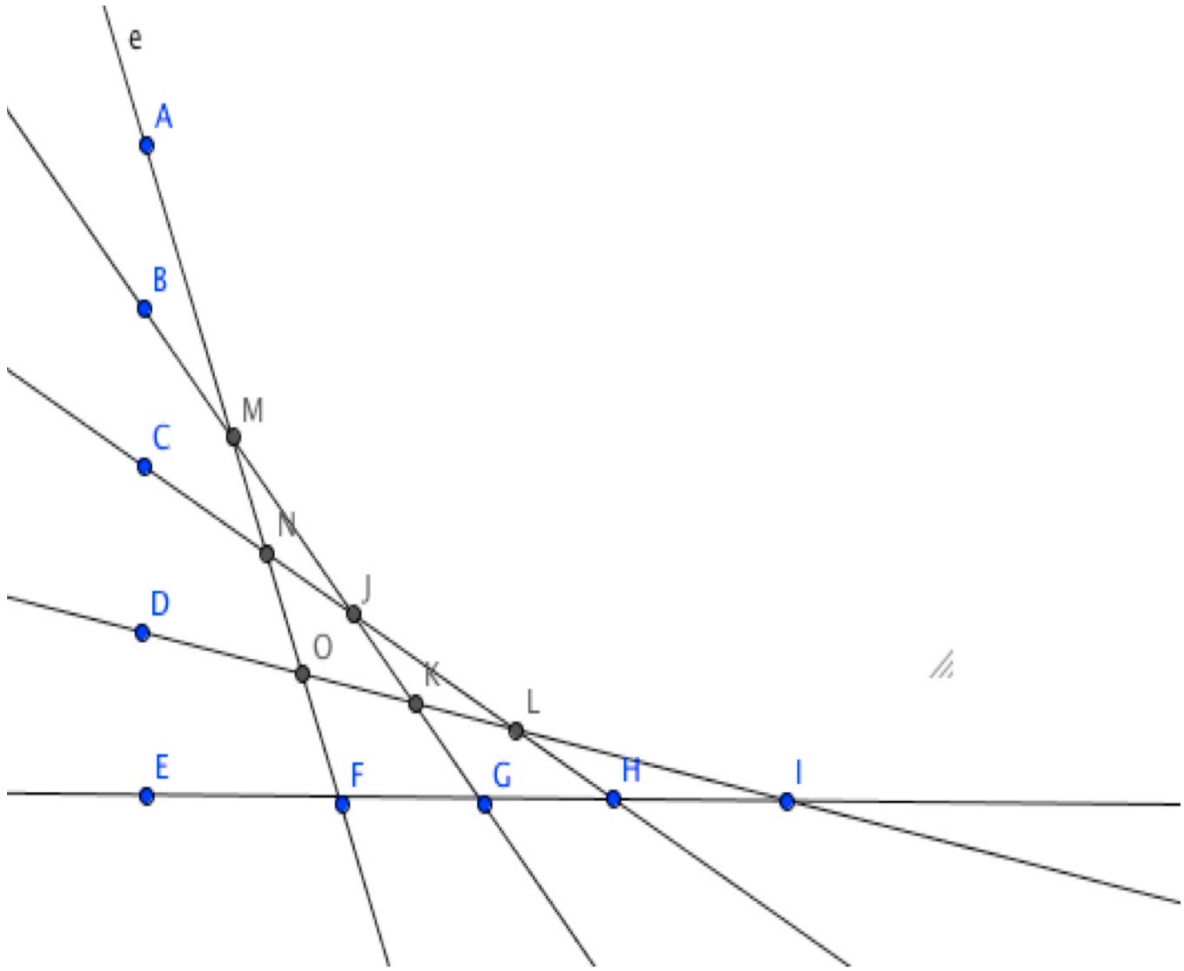
e
A
B
M
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N
J
D
O
K
L
E
F
G
H
I

	Elimination of (H,T)	Elimination of (H, T*z-1)
not gen.true and not gen. false	(0)	(0)
generally true (and, thus, not generally false)	(0)	Not(0)
generally false (and, thus, not generally true)	Not(0)	(0)

Protocol

The elimination is not zero

iff

the zero set is contained in a hypersurface of degree bounded by D

iff

ditto for the projection

iff

the statement is generally true (false).

Gröbner Covers

Botana F., Kovács Z., Recio T., Weitzhofer S. (2012). "Implementing theorem proving in GeoGebra by using various methods", Computer Algebra and Dynamic Geometry in Mathematics Education. CADGME 2012, Novi Sad, Serbia, June 22-24, 2012.

Work in progress: P. Janicic, I. Petrovic, M. Hohenwarter...

Conclusions

• ICMI Study: “School Mathematics in the 1990's” (Kuwait, 1986)

“even if the students will not have to deal with computers till they leave school, it will be necessary to rethink the curriculum, because of the changes in interests that computer have brought.
Let us mention here just three of them:
a) Algorithms, b) Discrete mathematics, c) Symbol manipulation.”

• 1996 ICME 8-TG19, *Computer-based interactive learning environments*, N. Balacheff-J. Kaput-T.R. http://mathforum.org/mathed/seville/followup.html

http://mathforum.org/mathed/seville/followup.html

Mathematical knowledge is intimately bound to its setting: knowledge placed/knowledge learned

Reification (Verdinglichung) of mathematical knowledge in computer-based learning environments, and accompanying enrichment of mathematical experience due to progress in interface design and knowledge representation (ie. internal structures)

- We need to rethink not only **how** to teach but **what** to teach...