Student Documentations in Mathematics Classroom Using CAS: Theoretical Considerations and Empirical Findings

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Abstract

Students face many linguistic challenges in mathematics classrooms in which use digital tools are used: Not only do the students need to use the mathematical language adequately, in addition to their everyday language, but they also need to master the technical language of their digital tool. Since the distinction between CAS syntax and non-CAS syntax seems to be empirically necessary but not sufficient when looking at students' documentation, there is a need for a qualitative analysis of different forms of language used in a mathematics classroom that uses digital tools. The theoretical framework of the study uses ideas introduced by Kant. On that basis, six different lexical categories are described and discussed of language students use when working with CAS. The qualitative analysis of an empirical example will provide a detailed description of students' use of language in two cases.

1 Introduction

There is a deep relation between mathematics and language. In mathematics, we deal with *assertions*, with *propositions* and with *concepts*:

Fermat's ***assertion****. The Pythagorean* ***proposition****. The* ***concept*** *of limit.*

The objects we deal with in mathematics belong to the linguistic inventory. These objects are elementary units that we use for exploring mathematical structures and that we need when formulating new conjectures and that we relate in mathematical proofs.

Mathematics is a language: This paradigm, to some extent, mirrors the linguistic turn in mathematics education starting about four decades ago. Today, there are influential contributions in mathematics education that relate to language in many senses (Pimm 1987, Morgan et al. 2014), not only with respect to the conceptual nature of mathematics itself but also regarding the use of language in mathematics classroom, in mathematical interactions or documentations. One of the areas where this plays a role in a practical sense in mathematics classroom is the use of language when working with digital tools, especially when looking at documentations. Students' documentations differ in various ways when working with digital tools (e.g. Ball 2003, Weigand 2013).

A student documentation: *I press menu-4-1-4.*

Do you understand this sentence? Would you consider this to be mathematical language? Does that seem appropriate for an adequate documentation to you? *Mathematics is a language:* When students document their mathematical actions and solutions, they always use a certain configuration of concepts, propositions and assertions. But: What is it to *do* mathematics from that decisive linguistic perspective and is *menu-4-1-4* an act of speaking mathematically? On the one hand, we need to consider a specific linguistic view on the use of language to approach this question. I will relate to the idea of using different registers in order to raise the question of how to specifically look at linguistic phenomena when students document their work in mathematics classroom. This linguistic perspective will give fruitful hints for the theoretical framework (section 3) in order to find linguistic categories that students use when documenting their work.

The discussion of *menu-4-1-4* as a typical documentation will show the need for a detailed description of students' language when working with digital tools. Existing research shows that indeed, there is a significant change in the use of language by the students when they have digital tools available in class (e.g. Ball 2003). These findings can not only legitimate some of the main questions of this paper, but, moreover, they raise the question of the relation of the normative reflection on language use and the empirical reality.

In this paper, I report on a study conducted on the language students in 10^{th} grade use when documenting their actions and solutions while working with a CAS. I will focus here on the context of early calculus and functions. Of course, this is a very narrow approach to the questions above. But the findings (section 3) constrain that there are linguistic categories that are specific to student's documentations when using digital tools. Therefore, I will describe results of an empirical study, which aims at reconstructing lexical categories to describe the language that students use in their paper and pencil documentations when working with a CAS. I will use a linguistic perspective, which relies both on the theory of linguistic registers (section 2.1) as well as on past research on students' documentations (section 2.2). Especially for the latter contributions, I will discuss the interrelations between empirical description of documentations and normative questions of what might be an acceptable documentation. Within my own theoretical considerations I will rely on some conceptual ideas that can be traced back to Kant (section 3.1) in order to develop a theoretical frame to empirically develop the categories for describing students' documentations (section 3.2). This discussion will transpose the familiar process of first normative then empirical approach in order to conduct a study where the empirical investigation is done first and the development of normative criteria is the second step. In this paper, I will report on the first aspect, describe six categories and discuss some empirical phenomena (section 4).

2 Language and documentations in linguistics and mathematics education

2.1 Theoretical discussion from a linguistic perspective: Registers and Codes

It is specific to mathematics that its meanings and conceptual relations can be formulated within a very distinct register: „Mathematics can be singled out, among other forms of human imagination and ingenuity, by the very specific linguistic register, in which its ideas are formulated“ (Winsløw 1998). Hence, learning mathematics can be seen as a process of lexical acquisition, of learning to use a certain formal register that differs from an informal or everyday register. This idea is fundamental to mathematics education research, and e.g. Pimm (1987) or Freudenthal (1991) show the importance and difficulties of moving from everyday to technical language in mathematics classroom.

Different registers and codes

From a linguistic point of view, mathematics as a discipline is an interesting subject of research because of its distinct register. That means that mathematical ideas are formulated with a unique corpus of vocabulary (e.g. Pimm 1987) and with a specific grammar and sentence structure (e.g. *Let f be a holomorphic function on D…*). Halliday defines a register as "a set of meanings that is appropriate to a particular function of language, together with the words and structures, which express these meanings. We can refer to a "mathematics register“ in the sense of meanings that belong to the language of mathematics (the mathematical use of natural language, that is: not mathematics itself), and that a language must express if it is used for mathematical purposes“ (Halliday 1978, 195). In this way, the mathematical register uses certain symbols, drawings and verbal expressions, is a human construction and "is applicable and necessary in many contexts, in which quantity and form need formal articulation" (Winsløw 1998, p. 20).

Besides the mathematical register and the everyday register, Schleppegrell (2004) has described the *school register*, which can be described by its decontextualized and abstract form.

These three different registers, the *technical*, *everyday* and *school register*, play a fundamental role when looking at students' use of language in mathematics classroom and they have gained much attention in mathematics education research (e.g. Prediger & Wessel 2011, 2013, Pimm 1987)

In sociolinguistic approaches, this shift between different languages is described as a form of *code switching* (e.g. Moschkovich 2007, Prediger et al. 2014). Although the concept of code switching is mostly used in multilingual research contexts, the idea of acquiring a new language was adopted to learning mathematics itself by Zazkis (2000), who emphasizes the importance of translating between natural language and the language of mathematics. Zazkis describes these transitions as "moving back and forth between the mathematical register and register of everyday English, that is, code-switching" (Zazkis 2000, 43).

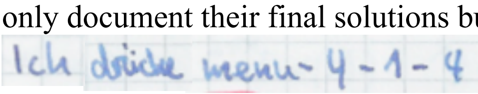
Using digital tools: How mathematics and language change

The discussion above shows that the scientific use of the conceptualization of registers is intensively used in mathematics education in two ways: To describe the transitions from everyday to mathematical register as well as to investigate phenomena in multilingual classrooms. I will now discuss how using digital tools not only changes the mathematical objects themselves, but also the language students use. There are questions that arise from this discussion that reflect on our understanding of the mathematical register itself.

There is a lot of potential in using digital tools in mathematics classroom. Not only can digital tools change the way we teach mathematics in class, in a way, digital tools can help to overcome "practices too much orientated towards pure lecturing or the procedural learning of mathematical skills" (Artigue 2002, 246). Moreover, the mathematics itself changes when using the digital tool. In the history of mathematics, the mathematical objects always changed when inventing or using a new tool in mathematics classroom. Especially when using digital tools, namely referring to CAS, the students do have direct access to different representations of a mathematical context and the tool may offer a dynamic approach to the concept that is being learned. In the context of teaching calculus in school, Lagrange emphasizes the change of the mathematical objects when using CAS, since "computers now offer a range of views (…) on a concept (…). For instance, the graphical utility is one between many views of the concept of function in a computer environment" (Lagrange 1999, 55).

But in the same way the mathematical objects themselves change because of the use of a digital tool, the language the students use also changes. As we see in the next section 3.2, important changes in students' documentations can be observed. One of the main reasons is, that by using the digital tool, the students are required to use some language of the tool that differs from both mathematical and everyday language. Another reason is that some mathematical algorithms are encapsulated in the CAS and are thus accessible without the need to actually expose the details of these algorithms to other discursive participants.

Let us take an example of a 10th grade student who is working on a task in which he is given a table with certain values that he graphs with his CAS. He is then asked to determine a proper regression function and to document his actions. He writes: "I press menu-4-1-4" (fig. 1A). This is a quite typical documentation when students work with CAS and are asked to not only document their final solutions but also their actions within the working process.



Translation: I press menu-4-1-4

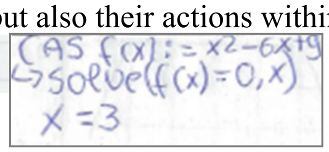


Fig. 1 A: The student documents the buttons being pressed. B: The student refers to the solve-command.

This documentation uses a certain kind of language that can be specifically traced back to the usage of the CAS. If the student had not used a digital tool, he would neither have had a need for an expression like the above, nor would he have had any basis to document something like *menu-4-1-4* in this context. But this documentation uses certain linguistic elements that the digital tool itself offers. Here, the reference to the button *menu* cannot be considered to be any kind of technical mathematical language. Instead, the word *menu* is a signifier for the button that the student pressed in the situation above. This becomes even more apparent when looking at the rest of the expression. Of course, by documenting *-4-1-4*, the student does explicitly not refer to the mathematical register, and hence we do not interpret this expression as a subtraction *-4-1-4 = -9*. Instead, the student refers to the buttons he pressed in order to generate the regression function. *-4-1-4* is a signifier to refer to the order, in which the buttons were pressed. This means that the expression *menu-4-1-4* can neither hardly belong to the typical linguistic inventory that the mathematical register offers, nor can it be considered to be some kind of informal everyday language.

In this sense, the expression *I press menu-4-1-4* does not fit to any of the registers above. I would not describe this expression as typically mathematical nor would I specifically consider this as an informal expression. Rather, this expression is very precise and serves a certain and an important function in mathematics classroom, namely to report on the students' action.

Therefore, this example raises the following question: *How does the language that students use change when they work with a digital tool?* From a linguistic perspective, referring to looking at specific registers, it is important to carefully shape out the linguistic categories that students use to show if there are specific words, expressions or sentence structures that are specific to the use of the digital tool. Referring to Halliday's (1978) notion of register, this requires a careful analysis of the use of the language when working with digital tools. This discussion also affects our understanding of the different registers themselves, since it becomes apparent that working with digital tools requires the students (and the teacher) to use a certain kind of language that would not normally be considered a match to one of the registers above. It remains an open question, if the notion of the *mathematical register* itself changes in the light of the expression *I press menu-4-1-4*. It might be possible to identify some kind of *tool register* in Halliday's sense as "a set of meanings that is appropriate to a particular function of language, together with the words and structures, which express these meanings" (Halliday 1978, 195).

2.2 Student documentations and the use of digital tools in mathematics education research

In this section, I will first draw on the main findings in research literature regarding the question of how student documentation changes when working with digital tools. It will become apparent that both the empirical question of what students write as well as the normative question of what students should write are of major concern. I will argue that the linguistic perspective will extend some of the insights we already have on students' documentations.

2.2.1 How documentations differ when using digital tools: the empirical layer

There are significant findings that report on empirical implications on student documentations when working with digital tools in mathematics classroom. For example, Ball (2003) reports on a study with grade 12 students (N=78) working with CAS (TI89, HP 40G, Casio FX 2.0) and *non-CAS students* (N=78) working with graphics calculators (same brands). Especially the qualitative analysis of the data shows that there is a remarkable shift within the language that students use when working with CAS in respect to working with digital tools in general. Ball & Stacey (2004) show that the students' documentations contain words that have a specific relation to the digital tool like "define", "equation", "substitute" (Ball & Stacey 2004, 91). They also report on the fact that CAS-solutions often contained the word "solve", while non-CAS solutions did not (Ball & Stacey 2004, 88). In addition to that, the CAS-solutions contain more function notation (e.g. $f(x)$) than non-CAS solutions (Ball 2003).

In a similar way, Weigand (2013) reports on a study where he uses documentations of German grade 12 students working with CAS in order to develop criteria for acceptable written

documentations. His empirical examples match with Ball & Stacey's findings since they show a use of expressions like "tanline($(x-2)^2+3$, x, 1)" or "main menu" (Weigand 2013, 2769).
These findings show, to some extent, how students‘ documentations change when working with digital tools in class. The examples show the impact of the digital tool on the documentations with respect to structure and language. On the one hand, there is evidence that the use of the digital tool affects the way, in which the process and the solution is documented, e.g. with respect to the length. On the other hand, students use the language that the digital tool offers. They explicitly refer to specific commands or to the menu of the digital tool.
From a linguistic perspective, it seems to be necessary to conceptualize this difference by categorizing the language. Ball (2003) proposes the following four features of written records:

Table 1: Codes for categories of written records (Ball 2003, p. 186)

Code	Written record
M	contains standard mathematical notation only
M'	contains some non-standard mathematical notation
W	contains one or more words that can be found in a dictionary
W'	does not contain any words that can be found in a dictionary

By making the distinction between standard (M) and non-standard (M') mathematical notation, Ball (2003) captures different linguistic phenomena that occur when students document their work with digital tools. For example, a documentation containing the expression *solve*($y-2=y^2-3$, *y*) would contain "CAS-Syntax" (Ball & Stacey 2004, 90) and therefore be categorized as M'. In this sense, the expression *solve*($y-2=y^2-3$, *y*) is considered as CAS-Syntax and is used as an example for non-standard mathematical notation.
Although this distinction between standard and non-standard mathematical notation is helpful, the two examples in fig. 1 illustrate why this distinction is not sufficient. They both show empirical examples of the empirical study that I report on in section 4. Both examples show typical elements of students' documentations when working with digital tools. In the example on the left (fig. 1A), student A is asked to document his actions while determining a regression function. He documents *I press menu-4-1-4*. In this documentation, student A documents the buttons that he presses on his CAS in order to determine the regression function. In the second example, student B is asked to solve the quadratic equation $x^2-6x+9=0$. Here, student B documents the command that is used: *solve(f(x)=0, x)*.
It is important to note that these documentations differ in fundamental ways although they both make use of non-standard mathematical notation. While student A refers to the specific buttons being pressed, student B documents a command being used. Hence, although the students both use lexical expressions that closely relate to the digital tool, these relations differ fundamentally. With repect to the lexical dimension of these two examples, the students use different lexical categories that both relate to the digital tool (see section 4). Regarding the mathematical dimension of the documentations, on the other hand, there is a second fundamental difference between both examples. While student A documents precise actions by referring to the buttons being pressed (*I press menu-4-1-4*), student B rather writes down some conceptual part of the solution, namely by referring to the idea of solving the equation $f(x) = 0$. In this sense, both documentations do also differ with respect to the mathematical conceptual performance. Both aspects, the lexical dimension and the distinction between different performative layers, will be initial idea of the theoretical framework in section 3.
By following the categorization of Ball (2003) though, both examples contain non-standard mathematical notation, namely *menu-4-1-4* and *solve(f(x)=0, x)*. Hence, they will be both categorized as M' following tab. 1. Since both documentations contain words that can be found in a dictionary (*menu* and *solve*), both solutions are also categorized as W following Ball & Stacey (2004). Hence, both documentations are categorized as "M' + W" in exactly the same way.

However, the examples in fig. 1 show that there are fundamental qualitative differences with respect to both the lexical dimension and the mathematical conceptual performance. Therefore, I argue, it is necessary to develop a more detailed framework in order to capture the differences described above. This will be done in section 3 by a two-dimensional category-system that makes a distinction between the mathematical performance of the documentation and the lexical-category that is used.

2.2.2 A question of adequacy: the normative layer

From a practical point of view there is an important question for teachers in mathematics classroom. What can be considered as *adequate or good* documentation? Of course, this is a highly normative question. That means all criteria refer to what is *counted as good* or what is *held to be understandable*. The question is bound by norms in such a way that there is no obvious objective criterion to judge if a given documentation is *good* or not. What we can observe, by looking at the empirical reality, is the way, in which students' concrete documentations meet the criteria we have. This question is relevant in several situations in class, especially in written tests. Weigand (2013) even shows that, in much documentation, there is no evidence if a calculator was used or not: "The problem for the teacher and the corrector of this examination is that the solution does not show whether and how the calculator was used." (Weigand 2013, 2771). He states that many students' documentations do not show if the calculator was used, although they are even very well reasoned. This is seen as a problem, because, in testing situations, the documentation of the way of how the solution was generated gives important insights into the mathematical actions and competences. Since, in written testing situations, the students' documentation is the only way to access these actions and competences and, furthermore, because students' language changed when working with digital tools, it is an urgent question to answer what might be accepted as adequate. This question is the normative pendant to the question of what students actually do document.
In this sense, Weigand (2013) pleads for "clear instructions for the documentation of written solutions" (Weigand 2013, 2772) and introduces normative criteria for student documentations. These criteria ask for understandable documentations that not only contain the expressions that can be seen on the screen, but also: "The solution describes the mathematical activities, it is not only a description in a special 'calculator language'" (Weigand 2013, 2772).
Ball & Stacey (2005) report on the project CAS-CAT, in which they also did research on how CAS influences assessment and teaching practices when CAS is used in secondary schools. In their project, the question of how students should document their work played a major role, both among the teachers that participated and among the researchers. It is one of the insights of the project that students seem to need "explicit guidance about what calculator language was acceptable in written work" (Ball & Stacey 2005, 119). Ball & Stacey (2003) develop the RIPA scheme that gives students a guide to structure their documentation by first documenting their **r**easons, then give **i**nformation about how they use the digital tool (e.g. documenting the commands), then document the **p**lan of their mathematical actions and finally give the **a**nswer to the task.
This scheme transports implicit categories (by offering helping means) of what is a *good* structure and a *helpful* way to initiate the process of communication by documenting one's solution. In this sense, this scheme transports (normative) criteria of adequacy and acceptability by offering a structure that should *help* students to *adequately* document their solutions.

2.3 Discussion and consequences

The approaches above, the normative criteria for adequate documentations (Weigand 2013) as well as helping means for students do document their work (Ball & Stacey 2003), face the

challenge of approaching the normative question of what could count as an acceptable documentation.
Still, some of the empirical examples above have shown a need to rethink some of the theoretical considerations that reflect on the relation of the normative and the empirical layer of students' documentations. I will point out two layers; the empirical layer and the theoretical layer.
With respect to the empirical layer, I draw attention back to fig. 1, where student A documents *I press menu-4-1-4*. This documentation does not meet the criteria of an acceptable documentation following Weigand (2013): this documentation is not understandable for others; there is no relation to any mathematical argumentation and a high affiliation to computer related language in an extreme case by documenting the buttons being pressed. On the other hand, this example raises questions in the light of the criteria discussed above: (1) While this documentation is not understandable for someone who does not know which CAS was being used, the documentation is highly understandable to the students' neighbour at the table and to the teacher: If we talk about understandability, *who* are we talking about? (2) Although this documentation is certainly not acceptable within final exams in year 12 for documenting ones actions, it is possible to think of situations in the beginning of a teaching unit, where students work on a complex task. These students might very carefully document their actions including the buttons being pressed in order to later remember how they created a certain solution, table or graph. In chemistry it is one of the fundamental tasks to precisely keep track of every step one does in the lab book. This may even include which substance is being put into the glass first and which second – which might cause severe effects when you change the order of filling the glass.
On the empirical level, there are situations, where students have a different intuition or opinion of what is an *adequate* way to document their solutions than many of us do when thinking about acceptable documentations. This aspect is of relevance. By formulating normative criteria of what could count as acceptable, we should consider that this question cannot be answered without looking at empirical data. Even more important, most of the criteria like those presented above transport a static notion of documentation categories. In the mathematics classroom, students develop mathematical skills as well as documentation skills. The latter means, a documentation in the beginning of a new learning unit is usually not on the same level of elaboration as it is (or as we expect it to be) at the end of the unit. As teachers, we not only initiate mathematical learning processes, but we also teach norms and skills for documenting the mathematical work. This is a central part of the learning process: *Mathematics is a language*. This is why normative criteria of what should count as an acceptable documentation should take this individual documentation development explicitly into account. In the next section, I will elaborate a theoretical framework for both formulating normative criteria with respect to empirical reality and with respect to individual development.
The second layer reflects on a *theoretical level* regarding the relation of normativity and empirical reality. Both the developed criteria and the means to structure the documentations face the problem that we can only show *a posteriori*, if certain documentation meets the criteria or the intended structure. There is no epistemological symmetry between the empirical reality and the normative categories. The only thing we can say is, if a given documentation meets the criteria or not. Moreover, this marks an epistemological gap because we can think of a situation, in which the documentation *I press menu-4-1-4* is *acceptable, necessary and understandable*. Thus, in the next section I will elaborate the idea of reconfiguring the relation of normativity and empirical reality by using some fundamental ideas that go back to Kant. The basic idea is not to place the normative criteria at the epistemological starting point and, from there on, ask *a posteriori*, if the students' documentations meet these criteria or not. The idea is to flip this process and ask which norms guide the students' documentations and then reconstruct these normative categories in the light of an empirical study. Although I will not an-

swer latter question of what is an adequate normative framework in this paper, I will give a theoretical notion of how this normative background might be developed in the light of the empirical reality.

3 Theoretical, empirical and methodological considerations

The discussion and the empirical examples above show that there is a need for research regarding the use of language when working with digital tools. I will address this question by developing linguistic categories within an empirical study (fig. 2). It is one of the key ideas of the overall project to develop normative criteria for adequate documentations by starting out with empirical research of students' documentations to reconstruct the lexical categories being used. By doing so, it is the aim to empirically analyze norms of student-documentations that are already implicitly in use. In other words, within this study, I do not start out with a priori criteria of adequacy for students' documentations. Instead, it is the aim to reconstruct implicit or explicit norms that students acknowledge by documenting their work and then to develop normative criteria in the light of these reconstructed norms. In this section, I will draw on some theoretical considerations about this aspect and I will do so by reflecting on some ideas that were put in by Kant (2011). Although, in this paper, I will only focus on the linguistic analysis of certain documentations, these results will give insights into some of the core features of the normative framework that develops out of this study.

3.1 Kant's idea on normativity and empirical reality

When students document their work, they must make many decisions. Looking at fig. 1, the students might take multiple implicit or explicit decisions concerning the following questions:

- How did I get my solution?
- What is the best way to document my process?
- Is writing down *menu-4-1-4* understandable? Is it acceptable? What will the teacher say?
- …

By documenting *I press menu-4-1-4* the student uses the words in the light of many implicit or explicit decisions that he made in reference to his conceptual action. This understanding of conceptual usage goes back to fundamental ideas by Kant in the 18th century. Kant initiated a fundamental paradigm shift regarding our view on conceptual acting. For Kant (2011), concepts have the character of rules that we follow. Applying concepts in this sense means to follow a specific conceptual authority that we obey. For Kant, concepts are required for perception; we structure our world with concepts. So whenever we apply concepts, we have to obey this conceptual authority and we acknowledge certain judgments being involved. This is what Kant refers to by saying that "thought is cognition by means of conceptions" (Kant 2011). This idea has been very influential in education, too. For example, Piaget draws on it by elaborating his idea of schemes (Piaget 1970).

Kant initiated a paradigm shift by turning the relation of experience and conceptual use: "all attempts to derive our concepts from experience and to attribute to them a merely empirical origin are 'entirely vain and useless.'" (Radford et al. 2007, p. 107)

In this sense, for Kant, understanding concepts means to understand the "rulishness" of concepts that means to know whether it is appropriate to apply a certain concept or not. In this sense, conceptual acting is highly normative. It was Kant's idea to push the idea of normativity that is already a fundamental part of our daily conceptual actions. For the contemporary philosopher Robert Brandom, it is one of the major tasks for modern philosophy to reconstruct this normative dimension of our conceptual acting: Kant „developed this insight in the form of a normative theory of concepts: judging and acting are thought of as applying concepts, where the concepts determine what we have made ourselves responsible for by having a belief or performing an action, the content to which we have committed ourselves. One of the central tasks of philosophy is to understand the normativity of human belief and agency." (Brandom 1999)

This fundamental Kantian idea is now being applied to the context of students' documentations. It is one of the general assumptions of this project that whenever we apply certain concepts, we have already made normative decisions and judgments. *Normativity is already in play whenever we communicate*. Hence, it is one of the major overall aims of the project to reconstruct and to better understand these norms. We have seen in (section 2) that when students work with digital tools, language changes significantly. Thus, it is one of the tasks of this study to work on the normative dimension that constitutes linguistic practices in this context *by empirically reconstructing norms*. It is a first important task to analyze the students' language when working with digital tools and further research is needed to reconstruct a normative framework for students' documentations.

Hence, I will introduce a two-dimensional grid that distinguishes between lexical categories and the mathematical performance (section 3.2). I conducted a study in order to empirically develop the lexical categories that students use. In line with Kant's ideas of conceptual acting, the normative dimension of students' documentations is a result of empirical research, a reconstruction of norms that are already in play when students document their work (bottom-up approach), instead of starting out this study with an a priori set of criteria of acceptance (top-down approach).

3.2 Two-dimensional analysis of students' documentations: Operationalization and data analysis

The empirical examples above and the theoretical considerations (Ball 2003, Weigand 2013) show the necessity for further research regarding students' documentations. The following two-dimensional grid for analyzing students' documentation was developed with respect to the theoretical discussions above. First, we need an analytical tool for analyzing students' documentations in order to shape out the lexical categories they use as a starting point of our study. This is one of the core ideas that can be traced back to Kant. Second, although it is useful to conceptualize this analytical study within the context of linguistic registers, we yet need a fine-grained analysis of the language being used in mathematics classroom, because language changes when working with digital tools. Third, although there are suggestions of distinctions between mathematical language and CAS-syntax, I have presented examples that show that although this distinction is helpful, it is not sufficient; there are empirical examples that show that it makes a difference if a student writes down *I press menu-4-1-4* in a final exam or in the beginning of a learning unit in class.

This last point shows that we both give respect to the lexical categories as well as to the mathematical performance. This is already the key idea of introducing two dimensions that are illustrated in fig. 2. The term *menu-4-1-4* for instance belongs to a specific lexical category, as we will see when looking at the results: the category *buttons*. This first dimension is named the **linguistic (lexical) performance** (see the columns fig. 2). At the same time, the term *menu-4-1-4* can be used by student A for different purposes, namely in order to describe a mathematical object (a), to describe the individual action of use of the CAS (b) or to describe the reasons for the choice of the specific digital tool or working mode, e.g. a graphical approach (c). This second dimension is named the **mathematical (conceptual) performance** (see the rows in fig. 2).

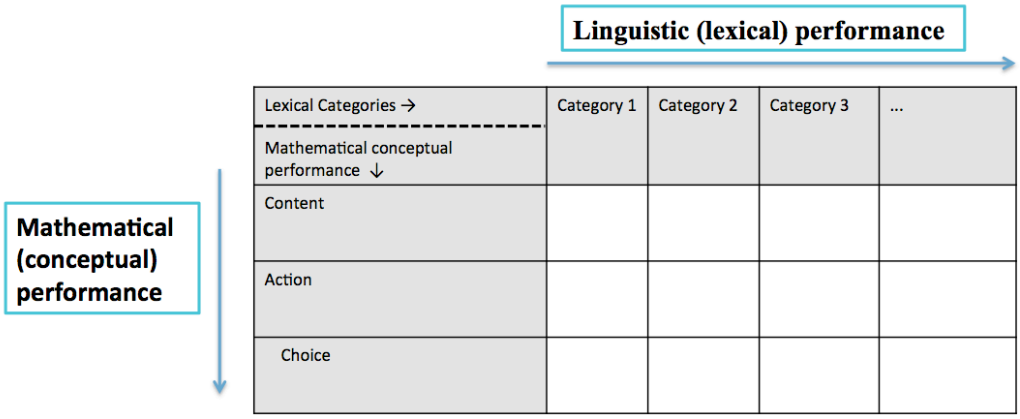


Fig. 2: Two dimensional analytical grid to analyze documentations

3.2.1 First dimension: The linguistic (lexical) performance

This first dimension of the analysis grid mirrors the exploratory character of the study. It is one of the aims to identify lexical categories that students use when documenting their work within an open coding process (Dörnyei 2007). These categories are generated within an open coding process by using the Grounded Theory (Strauss & Corbin 1990). It is one of the aims to generate categories in this part of the research process by comparing the data "against others for similarities and differences; they are conceptually labeled. In this way, conceptually similar ones are grouped together to form categories" (Corbin & Strauss 1990, 423).
In traditional linguistics, the term lexical category is used in a grammatical notion for describing parts of speech, like nouns, adjectives or verbs. In this study, I use the term lexical category to describe empirically generated and coded categories of language that students use when working with digital tools.
Looking at the example in fig. 1B, the solution is coded in two lexical categories. The student first writes down $f(x) := x^2-6x+9$. Here, he uses the symbol $:=$ for defining the quadratic function. Here, the student uses technical language that is also used in mathematical textbooks for defining. In this case though, the student uses the symbol $:=$ explicitly with respect to the use of the digital tool, because the definition of the function is one first step of his working process that is being documented here. In other words, there would be no need to use this symbol for this task if the student did not work with a digital tool. So although the symbol is part of the traditional mathematical register and hence seen as typical technical language, it shows a very distinctive reference to the digital tool and to the documentation of the mathematical content. Thus, this documentation is categorized *Technical language referring to the digital tool*. The categories that are generated within this study are introduced and discussed in detail (section 4). The student documentation is also categorized as *Command*, since it contains an explicit reference to the solve-command.

3.2.2 Second dimension: The mathematical (conceptual) performance

Besides the linguistic dimension, the idea of a "category of a word depends as much on how the word is used in discourse as on its conventionalized (lexical) meaning" (Payne 1997, 32). This is an important feature when analyzing students' documentations. As we have seen in fig. 1A, it makes a difference if a student documents the pressed buttons in a final exam or in the first lesson of a teaching-learning-unit. Since it might be necessary to document each step of the mathematical actions very precisely at the beginning of the learning process, this would

not be appropriate in a final exam when the student is asked to describe the mathematical actions.
Hence, it is important to distinguish the functions of the documentations the students use. In comparison to the exploratory character of the first analyzing dimension, the functional categories are theoretically derived. One of the main decisions for this study was to distinguish three different possibilities of *speaking about mathematics* (see also Neubrand 1990, 29):

1) *Documenting the content and conceptual ideas*: The documentation is used to communicate mathematical content, e.g. solutions, correctness of proofs or mathematical argumentation (c.f. Neubrand 1990, 29).
2) *Documenting the actions*: The documentation is used in order to communicate the individual actions within the process of working on the given problem, e.g. heuristic aspects (c.f. Neubrand 1990, 29).
3) *Documenting the choice of a certain digital tool*: The documentation is used to communicate the specific choice of the tool in use, e.g. by making explicit the advantages of working geometrically instead of solving the problem algebraically. In this category, students reason and reflect on the potential of a given digital tool. This category can be best described by what Cohors-Fresenborg (2012, p. 147) refers to as "*monitoring"* or documentation that is used to *plan, control and monitor* the working process with respect to the certain choice of the digital tool (see also De Corte, Verschaffel & Op't Eynde, 2000; Kramarski & Mevarech, 2003).

Note that for each of the different performances, it is possible to use different lexical categories. For example, the symbol *:=* may be used by students to document a certain definition (category 1: *Documenting the content and conceptual ideas*) or to document the action within the working process by first defining a function in a first step. Hence, it is one of the analytical tasks to give respect not only to the first dimension (lexical categories) but also to give respect to the task and the context, in which the language is used.

3.3 Data collection

The qualitative study was conducted with N=32 students in the 10th grade (upper secondary high school) in Germany. All students work with a TI NSpire CX CAS. The students worked on a paper and pencil test and could use their CAS to work on the problems. All problems were within the context of functional reasoning. The tests contained 4-7 problems each on 4 pages. Due to the limited space, I will give a brief discussion of the tasks of the test when discussing the empirical results of the study in section 4.
After collecting the data, each documentation was categorized with the help of the two-dimensional grid (section 3.2) in order to generate the different categories within an open coding procedure. Within this explorative study, it is the aim to empirically develop lexical categories from the data. Each piece of data was coded in two ways, namely with respect to the lexical category and the mathematical performance by using the following form:

lexical_category:mathematical_performance

For example, the expression *I pressed menu-4-1-4* was coded as *button:action*. The code *button:action* contains the two dimensions: The expression *I pressed menu-4-1-4* is categorized as *button* in the first dimension of lexical categories. Since the student documents his actions, it is categorized as *action* in the second dimension of the mathematical performance. This way, each documentation was categorized along these two dimensions.

The students all worked with a TI NSpire CX CAS on tasks about functional reasoning (section 4). Hence, this study reports on the following research questions:

- Which linguistic categories do students use when working with CAS on tasks about functional reasoning?

- Which phenomena can be analyzed when students document mathematical actions and mathematical results?

4 Results & Discussion

The first result will show six different lexical categories as a result of the empirical study (section 4.1). The second result reflects on two empirical phenomena that can be observed when looking at the lexical categories in detail (section 4.2).

4.1 The lexical categories

Table 2 shows the different lexical categories that were found within the empirical study. Together with the second dimension by defining three different layers of conceptual performance, it unfolds a two dimensional table. In this section, I will give a brief description and prototypical examples of the different categories. Therefore, I will give a brief definition and description of the categories and then describe some examples and observed phenomena in detail.

Table 2: The six lexical categories on the horizontal axis. Each documentation that was categorized as a certain lexical category was also coded with respect to the mathematical performance.

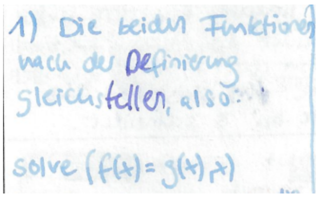
Lexical Categories → Mathematical conceptual performance ↓	**Command**	**Buttons**	**Menu / System**	**Math. symbolic expression**	**Technical language ref. to digital tool**	**New expression**
Content						
Action						
Choice						

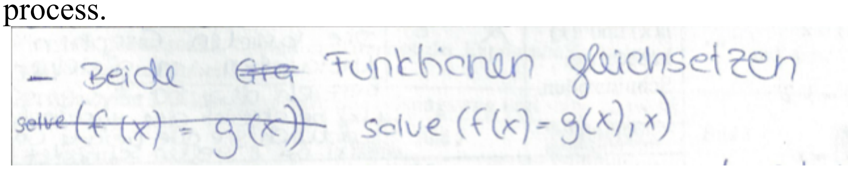
4.1.1 Category 1: Commands

Documentations that are categorized as c*ommands* have an explicit reference to the command being used. A *command* is what students can explicitly type in their calculator to initiate a certain mathematical process with their CAS.

Students often document an explicit reference to certain commands that they use when working with their CAS. In this sense, *solve, zero* or *binompdf* are categorized as commands, since that is what the students explicitly type in their CAS.
In this study, two cases of dealing with commands could be observed. In many documentations, students used the expression of the command to refer to it.
In fig. 3, the students work on the following task: “Describe 3 ways to determine the intersection points of the graphs of the following functions f and g with CAS: $f(x) = x^4 - x^3 - 5x^2 - x - 6$ and $g(x) = x^3 - 6x^2 + 6x - 5$.“ Here, the students did not necessarily have to determine the intersection points, but they had to describe different ways to determine these intersection points.
The student in fig. 3A uses the term *solve* to describe his actions. He refers to the solve-command by stating that he used the solve command within her working process. There is an interesting variant of this case when looking at the data in detail. In comparison to the first case in fig. 3B, the student in fig. 3A writes down: “Equating after defining the functions that means: solve(f(x) = g(x), x).” Note that the student uses *equating (German: gleichstellen)* instead of the proper term equalizing (*German: gleichsetzen*). The student explicitly writes down the CAS syntax that he typed in.

The use of the additional expression "*(f(x) = g(x), x)*" offers the students more expressional possibilities of documenting than only using the *solve*-command. While *solve* can be used to refer to the command being used only, the expression *solve(f(x) = g(x), x)* can be used for at least three different purposes: First, like in fig. 3A, the expression can be used to refer to the command itself. Second, it can also be used to document the mathematical idea. In this case, the student shows what she means by "equating": He starts out by equalizing the two functional expressions of f and g. This way, he can transport *a mathematical idea* by documenting the solve-command. But there is a third aspect of what the expression *solve(f(x) = g(x), x)* can be used for: It gives insights into the correctness and detailed steps of conceptual aspects of the working process. Fig. 3B shows the way, in which the student corrects his documentation, because *solve(f(x) = g(x))* does not lead to a sufficient solution. This way, by documenting what the student types in, it is possible for the teacher to get detailed insights into the working process. This is an important feature of the expression: It is possible to use it to communicate these details of the working process.





Translation fig. 3A:
1) Equating the two functions after the definition, hence:
solve(f(x) = g(x), x)

Translation fig. 3B:
- Equalizing both functions:
~~solve(f(x) = g(x))~~ solve(f(x) = g(x), x)

Fig. 3: Students documenting commands

4.1.2 Category 2: Buttons

A documentation is categorized as *button*, when students document buttons that they pressed during their working process.

For their documentations, students sometimes make use of buttons, which is another lexical category. A prototypical example in fig. 1A shows that the student gives insights into his working process by documenting *I pressed menu-4-1-4*. This documentation is categorized as *button:action*, since the student describes his actions by referring to which keys he *pressed*. Note that an example like *I chose the Graphs-menu* is not coded as *buttons*, but rather as *operating system* (see examples in the next section 4.1.3), since many documentations use a reference to the *CAS as an operating system*. Since there is a distinction between explicit buttons being pressed and the *operating system* of the digital tool, this is a different category.

The following examples show documentations of students working on a task on regression. They were given a table showing data of a growing plant culture over time. They were first asked to choose a type of regression that would fit to the context. After that, they were asked to determine a regression function and to document their working process.

Some interesting phenomena of students that documented their buttons within that task could be observed.

The qualitative analysis of students' documentations that used buttons shows a variety of different phenomena regarding five different types of notation that could be observed. The CAS being used contains a variety of different buttons: e.g. numbers, mathematical symbols, and an alphanumeric keyboard. The students make use of documenting a whole variety (see table 3).

Table 3: Documentational variants for buttons

	Notation-Variants of documented buttons	Example	Figure
1	Numbers	*Press* ***4-1-3***	4B
2	Signifier / word	*Press* ***doc****, then* ***enter***	4C
3	Concrete button with frame	*Press* ***menu***	4D
4	Mathematical symbols	*Press* ***squared bracket***	4A
5	Non-mathematical symbols	*Press* ***gap***	4A

The student in Fig. 4B uses numbers to refer to the number-buttons being pressed within the working process. Examples like these show that in mathematical documentations, the epistemic quality of numbers is used in different ways. That means that the student uses the expression *4-1-3* in order to communicate explicit parts of his working process. This term does not have a mathematical function within the act of communication. One has to give respect to the communicative situation, and if one did not, it could also be possible to infer that *4-1-3* would hint to a subtraction, meaning: $4-1-3 = 0$. Interesting about examples like these is the fact that the students are very aware of the communicative situation and attribute different meaning to (typically very common) mathematical symbols. The following example shows that fact: You get the regression function $f(x) = x^2 + 2$ *by pressing menu-4-1-2*. Here, $x^2 + 2$ and *4-1-2* would be usually considered as typical part of the mathematical register. But of course, *4-1-2* is not meant to be interpreted as part of the typical mathematical register since it refers to the buttons being pressed. The student and the reader attribute different meanings and functions to expressions that are usually used mathematically. But, in this case only one of the expression $x^2 + 2$ is actually documented with this mathematical notion.

4A: transcript & translation	*A und B für Jahr und Anzahl und dann eckige Klammern mit einer Lücke. Dann auf enter.*
	A and B for year and number and then squared brackets with a gap. Then press enter.
4B: transcript & translation	*Gehe auf Menu, dann Statistik (4), wähle 1 statistische Berechnungen und führe eine Exponentielle Regression (A) durch*
	Go to Menu, then Statistics (4), choose 1 statistical calculation and do an exponential regression (A)
4C: transcript & translation	Als nächstes drückt man "menu" (...) und drückt "enter"
	Then you press “menu” (…) and then “enter”
4D: transcript	Menu

Fig. 4A-D: Students documenting buttons

In fig. 4C, the student uses the inscription on the button to document which buttons were pressed. One of the main differences to the other examples above is the fact that these inscriptions use words that can also be part of everyday register (like *enter the door*). This example differs in a way, since the inscription is a regular word. There are differences regarding the semantic content that can be transported when using these words for the documentation. This shows the analysis of using the word *enter*, which the student uses to refer to the return-key on his CAS. In everyday register, the term enter can be used for example in order to express that someone enters the room. In this case, the word *enter* is used to express a certain semantic meaning. In analogy, the word *enter* in a mathematical context is used e.g. to express that someone *enters the values in the table*. Although we can differentiate the multiple expressive possibilities within each register, we can still see that this word is usually used as a regular part of the lexical body to express a mathematical idea when used in mathematical register and a non-mathematical idea when used in everyday register (see table 4).

Table 4: Mathematical register, everyday register, or none of the above? Documentations of buttons show the difficulty of using the familiar conceptualization of *register* to specify the language use.

Mathematical register	Documentation of buttons	Everyday register
I enter the values.	Finally, press enter.	They enter the room.

In fig. 4C though, the term *enter* is used as a proper noun to indicate which button was pressed. This word is not used in a standard grammatical context, since *enter* is a proper noun in this case. It is neither used like in regular mathematical nor in regular everyday register. Hence, by referring to the button in this way, the student uses new expressive possibilities.
An interesting variant of both cases above is when the student indicates that he refers to the button by framing the word (see fig. 4D), like *enter*. The frame indicates that the word enter is not used in a regular everyday or mathematical sense.
Finally, fig. 4A shows two examples of how students use and especially describe buttons with mathematical and non-mathematical symbols. The button *[ ]* is used to type in squared brackets, which is a standard symbol in mathematical notation. Hence, when the students document the use of this button, they have to attribute the (proper) term of how this symbol is used. Similar to that is the documentation of buttons that refer to non-mathematical symbols like the space-button. Like above, the students have to attribute the (proper) term when documenting its usage. Fig. 4A shows that the student uses the word *gap* (*German: Lücke*) instead of *space* (*German: Leerzeichen*) in order to refer to that button.
Important to note in this context is the fact that the buttons were exclusively used in order to document the students' actions. This can be explained by the fact that the explanatory power of documenting buttons can be traced back to the students' individual actions. Students usually use the reference to buttons either to back up their steps within the working process by referring to the menu and also refer to the certain buttons being pressed, see fig. 4B: *Choose statistics (4),* ***1*** *statistical calculations*. In this example the students indicate, which button they pressed in each step they went through the menu. In this example, the buttons *4* and *1* serve as backups, because the documentation would have been understandable without them. Also, buttons are used to document different steps of the work like in fig. 1A: *I press menu-4-1-4*.
This discussion of the results shows that students do not only use a variety of different documentations when referring to buttons. Moreover, we can find different expressive resources that each have specific expressive power and are used in specific communicative situations.

4.1.3 Category 3: CAS as an operating system

Documentations that are categorized as *CAS as an operating system (short: system)* contain expressions that explicitly refer to the digital tool as an operating system.

Note that documentations that are already categorized as *button* or *command* will not be categorized as *operating system*.
This category (3) can include references to the user interface (*Open the application* ***Lists & Spreadsheet***, fig. 5B), to certain operations (*Open a new* ***document***, fig. 5B) that students see and choose on the display. The example in fig. 5A shows the way in which students use the CAS as an operating system in order to describe mathematical actions like changing the scale of a coordinate system. In this case the student refers to changing the window adjustments when given a certain graph of an exponential function. This example in fig. 5A will be discussed in detail in section 4.2.

5A: transcript & translation	*Funktionen in Graphs eingeben, notfalls Fenstereinstellungen ändern und Punkte ablesen*
	Type in the function in Graphs, if necessary change the window-adjustments and read off the points.
5B: transcript & translation	*Ich öffne ein neues Dokument, öffne die Applikation Lists & Spreadsheet, benenne die eine Spalte xx und die andere yy*
	I open a new document, open the application Lists & Spreadsheet, label the first column xx and the other one yy

Fig. 5A-B: Students documenting references to the CAS as an operating system

4.1.4 Categories Mathematical symbolic expressions (4: math_sym), Mathematical expressions referring to the digital tool (5: math_dig) and New expressions (6)

Documentations are categorized as *Mathematical symbolic expressions* (4), if they contain symbolic expressions.

Mathematical symbolic expressions are an integral part of students' documentations from the beginning of their mathematical acting. A prototypical example is the description of the mathematical idea that is worked on with the CAS (*Equalize the functions with CAS:* ***f(x) = g(x)***). In this case, the functions f and g are used to document the initial idea of the working process. Note that whenever mathematical symbolic expressions were used as part of a command (*solve (f(x) = g(x), x)*), this was categorized as *command*.

Documentations are categorized as *Mathematical expressions referring to the digital tool* (5), if students document regular mathematical or colloquial expressions that refer to the digital tool.

In many documentations, interesting phenomena of regular technical mathematical language with reference to the CAS could be observed. One example of a student documentation using mathematical (but not formal symbolic) expressions with a strong reference to the digital tool is the following: *I had the graphs drawn*. This example shows the very slight but also very fundamental changes in students' language that can be traced back to the use of the CAS. In this case, the student documents that the graph was drawn by the CAS. It is not the student that draws the graph, but instead the graph is being drawn by the CAS. In examples like these, mathematical expressions change in a way that the CAS becomes an individual co-actor within the working process. Certain tasks are given to the CAS within this process. Hence, in the reflections of the documentations, the CAS has the autonomy to actually do certain steps of the working process that the student would usually do when working without the digital tool.

Documentations are categorized as *New expression* (6), if students make use of non-existing or artificial expressions that cannot be traced back to either the digital tool or the regular mathematical or colloquial language.

This last category contains new expressions that are not part of regular everyday or mathematical language. A prototypical example will contain a mixture of a German and English word (*Ich habe die Gleichung* ***gesolvet***). The expression *gesolvet* is a mixture of the German word for solved (***gelöst***) and the English word *solve*.

4.2 *The secret authority of concepts:* Two cases of documentating actions

In this section, I will describe two students' documentations in detail. For this analysis, I will use the categories discussed above. Also in respect to the theoretical considerations (3), I will show some specific phenomena of students' documentations when working with CAS.

The following documentations were produced when working on a task on functional reasoning. It was one part of the task to reflect on the question, how you can show that the two graphs of the functions *f1* and *f2* with $f1(x) = 3^x$ and $f2(x) = 4x+2$ have two intersection points. The students were given the screenshot from fig. 7.

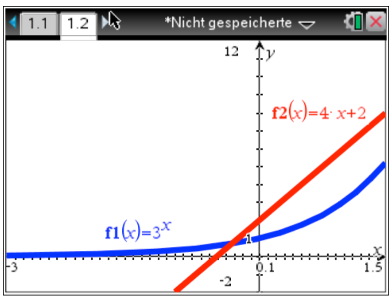


Fig. 7: Screenshot

I will now compare two different documentations on that task by the students Bill and Sandy.

Bill: „Type in the function in Graphs, if necessary change the window-adjustments and read off the points."

Sandy: "Change the scale of the representation."

Bill's expression "change the window-adjustments" was categorized as *system:action*, because the documentation contains clear references to the operating system (*window-adjustments*). Note that this documentation contains other expressions that were also categorized as *system*, here the expression "Graphs" as *system:choice*. The expression „Type in the function" was categorized as *math_dig:choice* and „read off the points" as *math_dig:action*. Sandys documentation was categorized as *math_dig:action*.
I will now focus on the comparison on the following two expressions:

Bill: change the window-adjustments (code: *system:action*)

Sandy: change the scale of the representation (code: *math_dig:action*)

This comparison gives some insights into the use of language when working with a CAS on functional reasoning. Both documentations refer to the same action, which is to see two intersection points on the screen.
While Bill refers to the operating system of his CAS by intending to change the *window-adjustments*, Sandy refers to the mathematics by intending to change the *scale*. Although both students refer to the same actions they intend to do with their CAS, they not only use different language, moreover they both refer to different epistemic references for describing their actions: CAS (Bill) and mathematics (Sandy). By analyzing these examples with the categories developed above, we can not only show that students use different lexical categories when documenting their mathematical actions. The detailed analysis shows that they refer to different epistemic references, precisely to the operating system and to mathematics. On the other hand, Bill's documentation contains no explicit reference to any mathematical idea or concept, whereas Sandy's documentation misses any reference to her technical actions.
The discussion above explicitly focuses on the empirical phenomenon of what the examples show. The theoretical considerations (section 3) showed the tension between the empirical and the normative dimension when looking at students' documentations in mathematics classroom using CAS. With respect of the theoretical considerations regarding the idea of developing normative criteria in the light of the empirical data, these two examples can reveal some important aspects. Therefore, we should look at the question of appropriateness of these two examples. The description of a mathematical action or content with explicit reference to the digital tool can be especially important or necessary in situations, where students learn any new concepts, where they act mathematically in explorative situations or where they deal with complex mathematical problems. These situations are often typical for learning situations. The argument is that we can think of situations, where documentations *should* consider the reference to the tool in order for example to memorize each step of the working process. On the other hand, especially in testing situations or in situations, where teachers would like the students to use consolidated and regular mathematical language, it would be appropriate and necessary to use standard mathematical expressions. This short discussion shows, in which way it is possible to reconstruct the normative dimension of language use in the light of the empirical findings: There are certain situations (e.g. especially focusing on the *learning process*), where we have a different normative surrounding than in others (e.g. especially focusing on using consolidated mathematical language). So although the question of developing a normative framework for using mathematical language when working with CAS will not be solved in this paper, the data, the analysis and the discussion shows the direction, in which the development of lexical norms can emerge out of the empirical findings. The discussion and development of normative criteria for the use of language when working with digital tools has to take into account the two different situations of learning and performing.
Looking at these two documentations from the perspective of the mathematical register, the *scale* (Sandy's documentation) is certainly something we consider to be part of that specific register. This is not the case for *window-adjustments* (Bill's documentation). That means that Bill uses a language that will not be considered to be regular part of the mathematical register, although he refers to the same action as Sandy does on the semantic level. Hence, the compar-

ison of these two examples also reveals the necessity of possibly rethinking what we would usually call the mathematical register, because Bill's example shows that *the change of window-adjustments* expresses his mathematical idea in order work on the task.

This discussion also shows the extent, to which Kant's ideas I have discussed in section 3 are fundamental to this analysis. Bill and Sandy use different words in order to describe their actions. In a Kantian sense, by choosing the terms *window-adjustments* and *scale*, Bill and Sandy apply different concepts to structure their actions, they obey different conceptual authorities and hence, by doing so, they are committed to different conceptual consequences. Or, to put it in Brandom's words: "the concepts determine what we have made ourselves responsible for by (…) performing an action" (Brandom 1999). But, in the cases above, this secret conceptual authority is certainly something that both Bill and Sandy are not aware of. I will conclude in the next section that it is one of the main implications of this insight that there is much potential and there is a need for making this secret explicit.

5 Outlook and Implications

In this paper, I have discussed results of an empirical study on students' documentations on two different layers. First, I have introduced empirically generated lexical categories of language that students use when working with a CAS on functional problems. Second, I have discussed an empirical example that showed some interesting phenomena when qualitatively looking at students' answers. I will now discuss some of the main features and implications of these results as well as the limitations of this study.

In section 3 I theoretically developed my understanding of talking about the normative dimension of students' documentations. I discussed some of Kant's ideas for my argument that the norms of what should count as a *good* documentation cannot be developed without looking at the empirical reality. I discussed a bottom-up approach: In a first step, the analysis of student documentations shows the lexical variety of students' documentations with respect to the mathematical content, action and choice of the digital tool. In the detailed qualitative analysis of one empirical example in section 4.2, it was possible to contrast language that refers to mathematics and to the digital tool. We can now ask, in which situations it is *appropriate* for documentations to refer to the digital tool and to the mathematics itself. For the students in explorative learning situations where they will carefully document and memorize each step of their working process, it is useful to document the reference to the digital tool. There are other situations, like in final exams, where this detailed description is not necessary. This way, we can empirically reconstruct normative frameworks for different situations. The development of this normative framework is one of the main tasks of further research. In the light of this normative framework, it will be one of the next steps to develop helping means in order to support students' documentations. Note that this approach does not start out with criteria and helping means to support these criteria in class in order to empirically validate, if these criteria and helping means have any empirical implications (top-down approach). Rather, the criteria and the normative framework are one of the goals of the empirical project (bottom-up approach). It is one of the main results though that we can distinguish performance and learning situations with respect to the use of language when working with digital tools. Each situation has its own normative framework regarding documentation.

One of the crucial aspects of this study relates to the special mathematical objects that the students were working with. The results not only show that it was possible to work out a variety of lexical categories that students use when documenting their work on functional aspects. Moreover, when describing their actions, students sometimes use three or more lexical categories in short answers in order to document their work. Still, these results are limited with respect to the mathematical content (functional reasoning). In the light of these results, it seems worthwhile to now expand the mathematical content and also consider geometrical constructions, for example.

Concerning the variety of lexical categories that students use, I consider it to be one of the main implications of this study to point out the relevance for actual classroom situations. The students use language that differs in various ways. The detailed discussion of the empirical examples could show that this also has major implications for the conceptual dimension of the documentations. It makes a strong conceptual difference, if students write about the mathematics or the digital tool. One first step would be to push a notion of *lexical consciousness* in class. That means to make criteria and different ways of documenting explicit to the students and discuss certain norms and the understanding of what makes a *good* documentation.

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