$\mathcal{M}$	Florian Schacht	CADGME 2014 - Title and Abstract	LU dortmund
intendrary and rescaled solicitation			
Student-Docu	umentations in Mathematic	Classroom Using CAS: Between Technical, Subject	ct-Based and Everyday Language
Students face adequately, ir become espee (e.g. Ball 200 for those writ e.g. emphasis means may h Since the dist students doct digital tools. This contribu work. Thereff analyzed	many linguistic challengee addition to their everyday (ally material when studen (3) that emphasize the exter then records (Ball & Stacey let the structure students' do use put on transitive que elp to structure students' do iniciton between CAS synt umentation, there is a need : ttion will present results of ore, different forms of docu	in mathematics classrooms that use CAS: Not only language, but they also need to master the technica is have to document their processes and their result to which CAS changes written records, and the ne 2005). In this context, there has been a focus on no tions regarding what might be an adequate docume cumentation (Ball 2003). x and non-CAS syntax seems to be empirically nec for a qualitative analysis of different forms of langu an empirical study that works out different categoris mentation using technical, school (subject-based) a	do they need to use the mathematical language I language of their digital tool. These challenges S. There have already been important results deed to learn to use the CAS syntax adequately mative questions on students' documentation -natation for tests (Weigand 2013) or which essary but not sufficient when looking at age used in a mathematics classroom that uses that students use in order to document their and everyday language will be descriptively
The qualitativ phases withir empirical ma problem solv	ve study was conducted wit n a school year, after recievi terial. Also, clinical intervia ing process. All exercises v	h 60 students in the 10th grade attending an upper s ng a new CAS, the students worked on paper penci ws were conducted in order to find out more about ere within the context of functional reasoning.	econdary highschool in Germany. In different I tests which served as a foundation of the the different uses of certain registers within a
florian.schacl	ht@math.tu-dortmund.de		



Filtratic County of the Schacht	technische universität dortmund
What is a good w	vritten record when using CAS?
1ch	bride menu - 4 - 1 - 4
Not adequate for written	tests!? Maybe adequate to memorize the individual actions!?
What we initially struggle wi	ith
- Documenting the pressed	buttons Do we think this is adequate?
- Communicating one's act	ions Do we understand menu $-4 - 1 - 4$ ?
- Reasoning mathematical of	decisions What is recorded here?
	3

EM	Florian Schacht	technische universität dortmund
des Vorlenzkinsnitt	I	
Aims an	d Structure	
1) Betw	een norms and fa	acts: What students should document and what they actually do!
2) Desc	ribing written lang	guage with categories
3) 2 em	pirical phenomen	a
4) Conc	clusion	

EEM Hadde is thereing actively des Mathematikanistes	Florian Schacht		t	U technische universi dortmund	tät
What	is a good v	written record	when using	CAS?	
Students acceptable	need "explicit gu e in written worl	uidance about what o	calculator language (Ball & Stacey 2005, 219	was	
Different,	but similar appr	oaches			
1) Guideli	nes of elements	of written records:			
RIPA: I	Reasons, Inforn Ball & Stacey 2003)	nation (including stue	dents' input), <b>P</b> lan a	nd <b>A</b> nswers	_
2) Criteria - "Not en - The sol - The sol	for Documenta ough to write d ution has to be ution describes	tions own, what's on the s understandable "for mathematical activi	creen! others" () ties" (Weigand 2013)	Tension betwee normative framev and empirical rea	en work ality
Ball 2003, Bal	& Stacey 2003, 2004,	2005, Weigand 2013			5







Index is Creating actively for Mehemological	Florian Schacht	technische universität dortmund	
Aims and 1) Betwe	Structure en norms and f	acts: What students should document and what they actually do!	
2) Descri	bing written lan	guage with categories	
3) 3 emp	irical phenomer	na	
4) Conclu	usion		
			9

Florian Schacht		technische universität dortmund
What do studen	ts write down when using	g CAS?
Observing written notations	3	
- Categorizing (i) mathema (Ball & Stacey 2004)	atical notation and (ii) words that can b	e found in a dictionary
Features of (CAS) solutio - Contain <i>,solve', ,define</i> - CAS solutions will be g	ns: '', <i>,equation'</i> generally <b>shorter</b> than non-CAS soluti	ons
Ich drid menu-1-1-4	$= \sum_{x \to 0} \sum_$	Distinction (CAS-language,
Same category (words	that can be found in a dictionary)	non-CAS language) is helpful, but not sufficient
Differences: procedure	e vs. solution, buttons vs. commands	
Ball 2003, Ball & Stacey 2004, 2005	i, Weigand 2013	10

Florian Schacht	t	U technisc dortmun	he universität Id
What do student	ts write down when using	CAS?	
Not easy to answer			
Two dimensions of c	conceptual practice in mathematics class	ssroom	
Mathematical (conceptual) - to describe a mathematic - to describe the individual - to describe reasons for the	performance: menu-4-1-4 can be used tal <u>object</u> a <u>ction</u> or use of the CAS te <u>choice</u> of the dig. tool	Idea: categ syste	2 dimensional orization m
Linguistic (lexical) performa - menu-4-1-4 as a specific lex	<b>nce:</b> xical category (buttons)	]]	

EM	Florian Schacht			tı	J techniscl dortmun	he universität d
Institut ta citoricute accuracy des Mathematikusencis		Ι	Lingu	istic (lexic	al) perfor	mance
		Lexical Categories → Mathematical conceptual performance ↓	Category 1	Category 2	Category 3	
Math (conc perfo	hematical ceptual) ormance	Content Action				
Aim of th	e study	Choice				
Explora	ation of lexical	categories students use v	when wo	rking with	n digital too	ols
for documenting the content for documenting their actions for documenting their specific choice of a certain tool						

EM	Florian Schacht		technische universität dortmund
hrstitut für Enseklung mit Frankrik des Mathematikunismöti	Т		
First dimen	ision: Mathematical	(conceptual) performance	- [-
1	ch driche m	enu- 4-1-4	Exponential Regression
Mathema	tical Content	math. object	
Mathemat	tical Action	procedure	
Choice o	f the dig. tool	process reflexion	
Neubrand 1990,	, Cohors-Freseborg 2010, E	Davidson & Pearce 1983	13

EM	Florian Schacht		technische universität dortmund
Second dim	ension: Linguis	tic (lexical) performance	
"Mathematics specific lingui	can be singled stic register in v	out, among other forms of huma which its ideas are formulated."	an imagination and ingenuity, by the very (Winslow 1998)
Register 1:	everyday langu	age	
Register 2:	technical langu	age	
Register 3:	school register	(Schleppegrell 2004)	
	Tool Register	: Different linguistic register?	$(AS f(x) := x^2 - 0x^4)$
	Research Qu categories do with digital to	testion: Which lexical students use when working pols?	X=3
Maier & Schweiger	1999, Morgan 2001, Pr	ediger et al. 2012, Zazkis 2000, Pimm 1987, Wi	inslow 1998, Halliday 1978

EM	Florian Schacht	tu tec do	chnische universität rtmund
Method - Que - Focu	ds stionaire(s) App. 200 studer using on functior	nts from schools with much and little CAS exp nal relations and calculus	perience
Example following following following following for a statement of the second	es  1) Describe 3 wa uctions with CAS  2) Bernd works f(x) = e <sup>x</sup> and of	by to determine the intersection points of the $f(x) = x^4 - x^3 - 5x^2 - x - 6$ $g(x) = x^3 - 6$ on the following task: How many intersection a linear function have?	graphs of the $x^2 + 6x - 5$ points do the
$\frac{1.1}{(x)-e^{x}}$ $\frac{1.1}{g(x)-ax+b}$ $\frac{1}{g(x)-g(x)}$	Picture 1	Image: Starger models.         Image: Starger models.<	Describe his solution. How could he show, that there are 2 i.p. max.?
He answ	veres: They hav	e one intersection point.	15

E	М	Florian Sch	acht				tu technische dortmund	universität
hretikus für Stranding des Wattemationstrad	untifernant da	First re	esults					
	Lexical Categ	ories →	Command	Buttons	Menu / System	Math. symbolic	Technical language	New
	Mathematica performance	Il conceptual ↓			System	expression	Tel. to digital tool	expression
	Content		1					
	Action			Λ			Λ	
	Choice	/	/ [					
	First, I press Menu $\rightarrow 6 \rightarrow 4$ .My approach: f(x) = f`(x)Er hat sie nach x gesolvet.					hat sie nach x olvet.		
Berno there of sol	d was wro are 2 x's lve(f(x) =	ong, becau in the sol g(x), x	ution	beause y increase i	you can so n <mark>Data &amp;</mark>	ee linear Statistics	First, you have to c then you enter the you have to label the	reate a table, values. Finally ne columns.

Testar in Execution	Florian Schacht	technische universität dortmund
Aims and 1) Betwe 2) Descri 3) 3 emp	Structure en norms and fi ibing written lan irical phenomer	acts: What students should document and what they actually write! guage with categories
4) Conclu	usion	
		17

EM	Florian Schacht		technische un dortmund	iversität
des Varhenskussenti des Varhenskussenti	enomenon 1			
Describi	ng my actions – Ł The Role of nat	out how? uralistic describing varieties		
First, yo Finally	ou have to create a you have to label	a table, then you enter the values. the columns.	(technical language : action)	case 1
- "Natu - Refei	rring to the repro	lary: <i>create, enter, label</i> esentation: <i>table, value, colum</i>	n	
Draw t	he graph and dete	rmine the intersection points!	(technical language : action)	case 2
- Math - Refe	ematical vocabu rring to the math	ulary: draw, determine nematical concepts: graph, inte	ersection point	-
Usefu	ul for documenti	ng each mathematical step		18

E	Florian Schacht		technische un dortmund	iversität
institut für Entwick des Mathematikunti	Phenomenon 1			
	Describing my actions - The Role of n	but how? aturalistic describing varieties		
	First, you have to create Finally you have to lab	e a table, then you enter the values. el the columns.	(technical language : action)	case 1
	Situation for applying	such a description:		
	Exploration	- memorizing each (technical) s	step of action	
	Important for the lea	rning process		_
	Draw the graph and de	termine the intersection points!	(technical language : action)	case 2
	Situation for applying	such a description:		
	Test – desc	ribing each (mathematical) step	of action	
	Important for testing	situations		19

E	М	Florian Schacht		technische dortmund	universität
kretitut für Entwick des Mathematikunt	Ph	enomenon 2		113 12	12 by
	The secr	et authority of co Two cases of do	ncepts ocumentations of math. content	1 intersection point?	n(d-3 <sup>x</sup> 2 21 12
	Type in read off	the function in G the points.	raphs, if necessary change the wi	ndow-adjustments and (system : action)	case 1
	- Doc	cumentation: ref	erring to the system / menu of	the CAS	
	Change	e the scale of the r	representation.	(technical language : action)	case 2
	- Doo	cumentation: ref	erring to the mathematical obj	ect / content	
					20

EM	,	Florian Schacht				tu techn dortm	ische universität uund	
The second secon	Phe secre "tool" actio (mer miss	enomenon 2 et authority of cc Two cases of do window-adjustm " register en refers to the nu) of the CAS ing references	ncepts occumentations nents system to	; of math. cor	itent - ma - act cor - no	1 intersection point? scale of the repre thematical regis ion refers to a r ntext references to th	sentation.	
Descrip	math	of math content	epts / ideas		act	ions		1
1) with 2) with	n refe	rence to the tool rence to the mat	itself (system, nematical cont	menu, softwa ext	are)	the learning p	orocess ons	21

EM	Florian Schacht		technische universität dortmund
des Nationalisations des Nationalisations des Nationalisations	enomenon 3	I	Task
The mor	e the better? In between difi	ferent lexical categories	How did you do the exponential regression?
Tech. Langu At first, I h	age:action ave generated	a table, then I have entered the	values and labeled the columns.
buttons:acti Then you Then you you press (3) (5)	on press "doc". press "menu". "enter"	-os the enter, wird eine J erdin die Werte eingetragi	delle erstellt, stann en emel die Spolten nuesen
System:ac →"fill in" →Graph →scatte	etion → graph plot	mennet werden. ann driekte men "de" z ein finet sich ein Hoordina Lied men "mein" - Graph Kei tet gibt mein ein zrie ma richt", anter	fügen » graph, nun Versystem. Als nächstes jugale (s) wir Streuchagtam (s), m die Ipaltengewannt bet wee

EM	Florian Schacht			tu	echnische universi Jortmund	tät
des Vorbenzeitungen der Vorbenzeit des Vorbenzeitungen der Ph	enomenon 3	I				
The mo	<i>re the better?</i> In between diff	erent lexical categories				
3 lexical ca	ategories to desc	ribe math. activity				
Tech. Lan	iguage					
Buttons	- Im - Ne	portant features of proces eed for conscious use of c	ss-deso differen	criptions t lexical cat	egories	
System		Tech. language	Syster	n	buttons	
		Exponential regression, because of typical shape of scatterplot	Menu →Scat	→Graphs terplot	menu" → (3) → (5) → enter	
Description	is of math. actions			Important f	for	
<ol> <li>with ref</li> <li>with ref</li> <li>with ref</li> </ol>	erence to the tool ference to the butto ference to the matl	(system, menu, software) ons hematical context		the learni the learni testing si	ng process ing process tuations	

EM	Florian Schacht	technische universität dortmund
Institut für Entwählung anderson) des Mathematikunsendti	Т	
Aims and	Structure	
1) Betwe	en norms and fa	acts: What students should document and what they actually write!
2) Descr	ibing written lang	guage with categories
3) 3 emp	irical phenomen	a
4) Concl	usion	
		24

EEM	Florian Schacht		tu	technische universität dortmund
Ba	ck to Kant – Summary	and outlook	]	I. Kant
- Stu - - -	dents use a lexical v Descriptions of acti Descriptions of con Consciousness of le pretative Layer	ariety when working on ( <i>representation</i> – tent ( <i>tool register – r</i> exical usage (3 cates	with CAS content) nath. register) gories)	Identifying the norms of "conceptual reality"
- vi Norm	sible distinction for s 1) the language 2) the language native Layer	tudents between situ refers to the tool (eg refers to the mather	uations, where it is imp g. procedures) (learnir natical object (testing	portant that ng process) and situations)
- 1	Need for criteria for t 1) learning process	he use of tool- and to (cumulative)	echnical register 2) testing situations	Approach normative questions and develop helping means



<u>g</u> v <sub>i</sub>	Florian Schacht	Begriffsbildung im Mathematikunterricht	tortmund technische universi
iding utitedan katerda			
Literatur: Ball, L. (2003 examinati: Ball, L., & Ste Fey, A. Cu, Reston, V. Ball, L., & Ste & A. B. Fu PME. (v0) Ball, L., & Ste Conferenc, Conferenc, Conforenc, Conforenc, Conforenc, Conforenc, Conforenc, Haliday, M., Kart, I. (2011 Maier, Herma Verlagsge Morgan, Can Teaching, Neubrand, Maier, Herma Verlagsge Morgan, Can Teaching, Schleppegrel Weigand, H Proceedin	b). Communication of mathem on question. The International tops, C. GUOJ, What should be the Communication of the Communication of the Communication of the Communication of the Communication of the Communication of the Communication of Communication of the Communication of Communication of the Communication of Communication of the Communication of Communication of Communication of Communication of Communication of Communication of Communication of Communication of Communication of Communication of Communication of Communication of Communicat	atical thinking in examinations: Features of CAS and non-CAS stud Journal of Computer Algebra in Mathematics Education, 10(3), 183 https://www.communecester.com/communecester	ant written records for a common year 12 1-194. 1-194. 1-194. 1-194. Mathematics Education (sp. 289-303). ecords with CAS. In M. Johnsen Heines, of Mathematics Education, Bergen: (Eds. ). Proceedings of the 29th leibourne: PME. der Mathematik education. 196. 142 – 45 197. 142 – 145 197. 145 1