An activity-theoretic approach to multi-touch tools in early maths learning

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In this article we present an activity theory based framework that can capture the complex situations that arise when modern technology like multi-touch devices are introduced in classroom situations. As these devices are able to cover more activities than traditional, even computer-based, media, we have to accept that they take a larger role in the model of interactions. We reflect this fact by moving the artefact into the center of our observations, leading to an artefact-centric activity theory (ACAT).

The theory was developed in the need of analyzing learning environments for primary maths education. We will start by showing an example of such a learning environment as well as the technological approach to it. Next, we will develop our framework that is based on Engeström's structure of human activity systems (Engeström 1987). We will detail how the different interactions correspond to aspects from mathematics education, programming, human-computer-interaction and pedagogy. In particular, we can isolate the process of instrumental genesis (Artigue), the different levels of activity theory in internalization and externalization mediated by the multi-touch environment, social interaction in group learning in the view of instrumental orchestration (Trouche & Drijvers) and design principles for computer-based tools (Ladel).

1. Multi-touch environments and the ACAT theory

In the last years the human-computer-interface has been evolved from indirect manipulation via a keyboard or mouse to direct manipulation using touch-sensitive interfaces. In fact, through the introduction of devices like the iPhone or, more recently, the iPad, we see a rapid adoption of multi-touch-interfaces in all age groups, including primary school students.

Multi-touch technology can capture multiple touches on a screen and convert these into actions into *events* that can be interpreted by appropriate software. In the simplest case this might be mouse actions like mouse clicks and mouse drags, but due to the fact that several touches can be combined into *gestures* it is possible for the user to give more information than just translations (that correspond to dragging the mouse) or (x,y) positions (that correspond to clicking the mouse). With two fingers it is easily possible to rotate or scale – or move several objects at once, each in a separate direction.

It is obvious to use such technology for young children in technology enhanced learning environments: (1) The user interface is easy to understand (sometimes even *natural*) and does add unnecessary complexity to the learning process; (2) The direct manipulation enables children to work with virtual manipulatives directly instead of being mediated through another input device; (3) It is possible to create environments with large screens (like multi-touch-tables) that encourage collaborative learning and communication of the children.

In order to create such learning environments it is helpful to invoke an educational design pattern. In our case, we would like to create a constructivist environment that enables children to move between different representations of numbers. They should be able to work enactively with the virtual manipulatives, those are presented in iconic form, and – depending on the activity – can be transferred automatically either simultaneously or on demand into symbolic representations.

An inherent problem of multi-touch environments is their enormous flexibility. By focussing on direct manipulation via multiple touches we can specify the *input channel* of our environment. For the visualization, the *output channel*, we must choose iconic and symbolic forms that are suitable for representational transfer. Using a multi-touch programming environment (Richter-Gebert & Kortenkamp, 2006) we are able to specify the connection between the *input channel* and the *output channel*. This program, together with the multi-touch hardware creates an artefact that mediates between the actors (the children) and the objects of doing (the virtual manipulatives). The children can only manipulate within the limitations set by this instrument (Engeström 1991).

> *"Studying the changes that learning environments undergo when technology-based artifacts are introduced means analyzing how activity changes as consequence of tools' introduction and how this change is meaningful for the students and the teachers."* (Bottino & Chiappini, p. 841)

The inherent complexity of an environment based on multi-touch calls for a theory that is able to guide us in analyzing it. Because multi-touch seems to be the central concept in our setting, we move it into the center of our theory, artefact-centric activity theory (ACAT).

The diagram below, adapted from (Engeström 1991), shows the resulting interrelations.

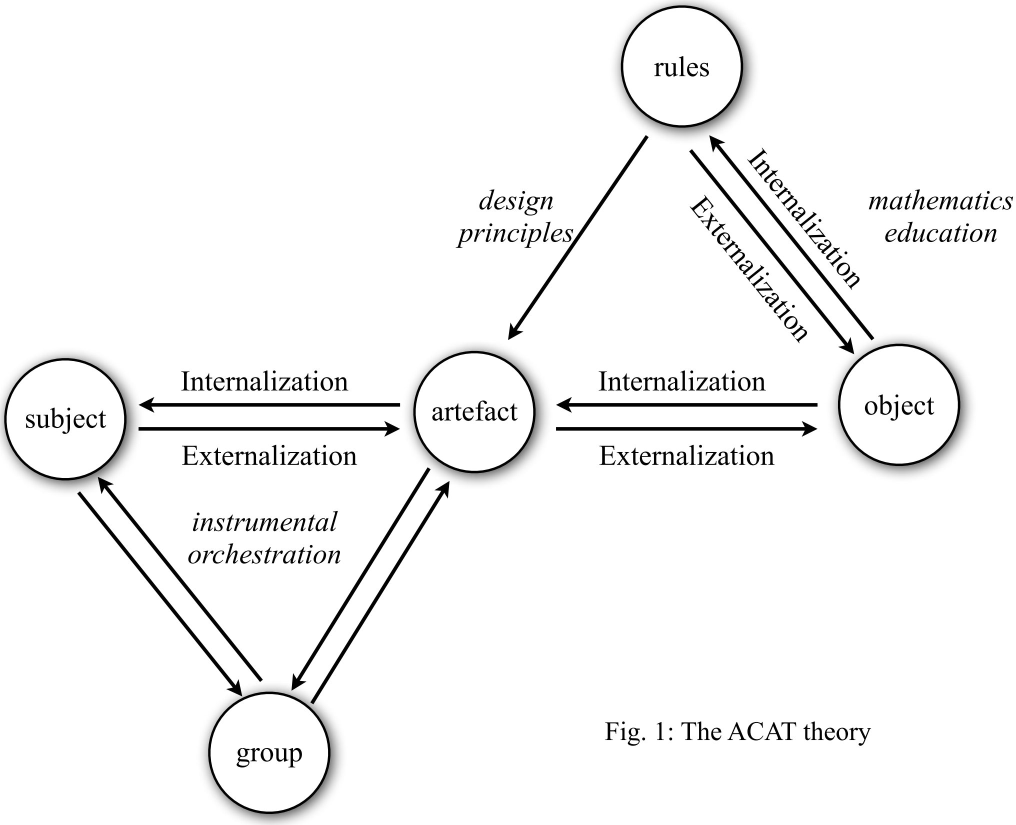

Fig. 1: The ACAT theory

The main axis of interaction is along the subject–artefact–object line. A subject, here the student(s), externalizes its concepts regarding an object (in our case: numbers) via an artefact (the multi-touch environment with its virtual manipulatives). The artefact itself externalizes the object through a suitable representation and visualization. The object is encoded into the artefact: the artefact is limited to the object's properties and aspects. The essence of being a number determines the artefact's behavior. Through manipulating the artefact, the student can experience the "numberness" mediated by it.

The role of mathematics education is to devise the rules that lead to the design principles used for creating the artefact. Mathematics and mathematics education traditionally create models for abstract objects together with rules to work with these models. Through the rules we define the object formally (externalizing it) and the rules are made to capture the nature of the object in the best way possible (internalization). Once such a set of rules is available, we can derive the discipline specific design principles for creating artefacts, that add to the general design principles from psychology and multimedia design (Mayer 2005).

A multi-touch-table is particularly well suited for working in groups. One factor is the size of the table as compared with a usual PC or laptop screen. However, this could also be achieved by projecting the display to a larger screen. More influential is the possibility to work *at the same time* as the students do not have to take turns with the mouse. It is hardly possible to fight for the input device if this is available to everybody at the same time. It has been shown that bullying effects (high achieving students take over in an activity by occupying the mouse) and non-subject specific communication that involves agreement discussions about “who is next” are drastically reduced in multi-touch environments, and students engage more in fruitful communication that is close to the topic (Harris u.a. 2009). Therefor, the group arrangement must be taken into consideration. We will detail this in the section on instrumental orchestration.

While a multi-touch-artefact seems to be an *enabling* technology as it provides lots of possibilities to work with virtual manipulatives that can act in numerous ways, we must stress that it is also, if not primarily, a *limiting* technology. We want to restrict the students’ externalizing actions to support the internalization of specific properties of the objects in consideration. Thus the mediation through the artefact is characterized by restriction and focussing.

2. Conceptualizing mathematics knowledge through the creation of rules for design principles and implementation of artefacts in ACAT

For the analysis and further presentation of our theory we concentrate on the upper right triangle of the ACAT theory from Fig. 1.

In Fig. 2 we replaced the generic artefact and object with two concrete examples, the multi-touch table MTT and numbers and operations, our object of interest for teaching and learning. The externalization is handled by visualizing numbers (for example through tokens, symbols or by locations on the number line). The internalization has to be done through programming the multi-touch environment suitably. We will give an example to illustrate this two-way process.

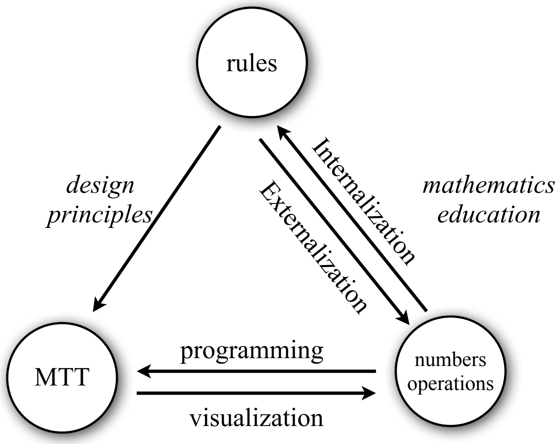

Fig. 2: The upper triangle

The number 8 can be seen as the sum of 5 and 3. An artefact that is used for working with numbers and considers the part-whole-concept as a fundamental principle of numbers that should be reflected in its behavior must be able to split 8 grouped tokens into two parts, say, 5 tokens and 3 tokens that are still in groups. This operation must keep the operation history, that is, the 5-group and the 3-group must “know” that they originated from the 8. A student asking for a representational transfer of the 5 and the 3 into symbolic form should receive the information that 8 = 5+3. The programming of the environment has to store this process information. The part-whole-concept becomes manifest in the coding of the artefact.

Using fingers and finger symbol sets is a common strategy in early maths. It is possible to transfer this into rules for designing MTT environments. For example, the “power of five” (Krauthausen 1995), that is using groups of five or ten to support the quasi-simultaneous recognition of quantities, tells us to group tokens automatically in groups of five or to offer an easy way to put five tokens at a time on the table (Ladel & Kortenkamp 2009). We sketch the underlying theory from mathematics education in the following section. We emphasize that due to the ACAT theory we can pinpoint the essential areas of investigation and see exactly where programming, visualization, design and the interplay with mathematics education are located.

2.1. Part-whole concept and Finger Symbol Sets

Many difficulties that certain children have in learning arithmetic arise from the fact that their concept of numbers and operations is only insufficiently developed. There exist two main concepts of numbers, the ordinal and the cardinal. The ordinal concept is the first concept children acquire. In this concept the verbal number row is understood as an ascending ordered row, in which every number has its fixed position, e.g. the number 7 means the 7th position in the number row (s. Fig. 13left). If a child does connect the last called number not only with the last object but with the whole quantity, it has developed an understanding for the cardinal concept (s. Fig. 3 right).

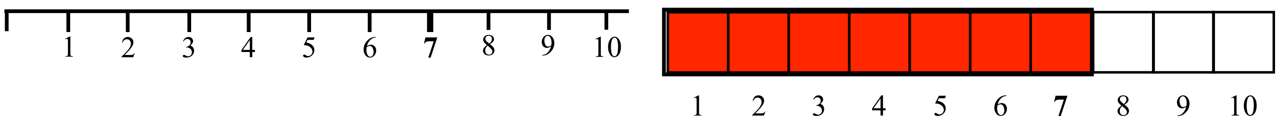

Fig. 3: the ordinal concept (left): numbers as position in the number row and the cardinal concept (right): numbers as a quantity

The part-whole concept is a a further development of the cardinal concept. Numbers are now seen as a whole of smaller parts. With this concept it is possible to compose a quantity, e.g. 7, in the different parts, e.g. 5 and 2 or 4 and 3 etc.

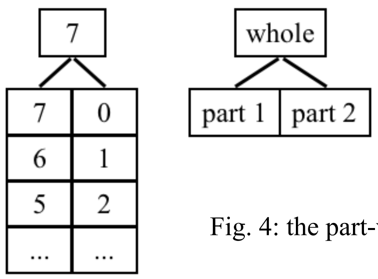

Fig. 4: the part-whole concept

The part-whole concept of numbers is already a first, a static concept of addition and subtraction. With the number triple whole - part 1 - part 2 it is always possible to formulate four equations:

$$W = P1 + P1; W = P2 + P1; W - P1 = P2; W - P2 = P1$$

As we can already see the part-whole concept accentuates the complementarity of addition. It is also essential for further learning of mathematics:

> *“The protoquantitative part-whole schema is the foundation for later understanding of binary addition and subtraction and for several fundamental mathematical principles, such as the commutativity and associativity of addition and the complementarity of addition and subtraction. It also provides the framework for a concept of additive composition of number that underlies the place value system.“* (Resnick *et al*., 1991, p. 32).

If a child is using the ordinal concept of numbers for addition and subtraction, those operations has the only meaning of counting forward or backward in the number row. This may work with smaller numbers but leads to problems at the latest when the number range is higher than 20, e.g. 47+52 or 82-63. Hence it is very important that a child performs the transition from the ordinal to the cardinal concept and develops the part-whole concept of numbers. We think there is a way to enhance the development of the part-whole concept by using Finger Symbol Sets (cf. Brissiaud, 1992). The use of fingers has a very bad reputation because they are often used in the way tagging one by one. This way equates to the ordinal concept of numbers. Finger Symbol Sets means the use of fingers in the sense of the cardinal concept. In this way we can benefit from the natural decomposition of the fingers in fives and tens. This is an important fact related to the ability of quasi-simultaneous acquisition of numbers. Quantities bigger than four can be identified quasi-simultaneously by composing smaller parts. E.g. "6" is presented by one hand of 5 fingers and 1 single finger. As researches show (cf. Ladel, 2011) children are able to recognize quantities that are represented by fingers very quick by relating them to 5 (one hand) or to 10 (two hands / one person). To the question why they are able to tell the quantity of shown fingers so quick they answer e.g. *„Because it is five and three."* or *„Because it is ten minus one."*

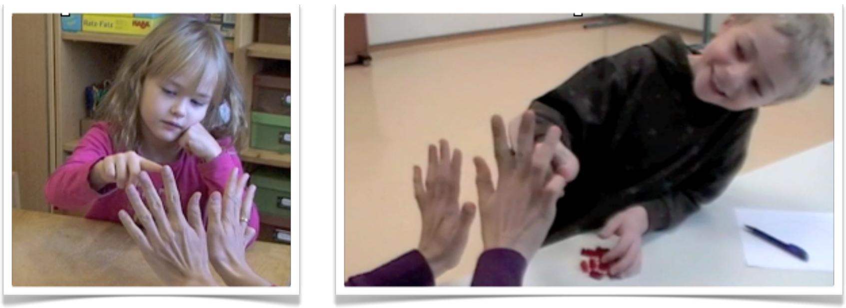

Fig. 5: counting word tagging to number (left) and Finger Symbol Sets (right) with 9 = 10 - 1

One task that we offer to the children to support the development of the part-whole schema is the presentation of numbers. They have to produce tokens by putting their fingers on the multi-touch table. In non-multi-touch learning environments the children have to produce the tokens sequentially or via a direct symbolic representation. In our approach, they are able to present a quantity like "seven" by putting seven fingers on the table – all at once. This is an advantage of computer-based learning environments that leads the children to compose quantities not one by one but in bigger parts (cf. Ladel & Kortenkamp 2009). As researches show it is a good way to convince children that this is the better way by writing the equation of producing the quantity, e.g. 6=1+1+1+1+1+1 is much longer than 6=5+1.

As it does not matter which of its fingers a child uses, this activity supports the process of abstraction (cf. abstraction principle (Gelmann & Gallistel, 1978)). To present the quantity „2" you can use the thumb and the forefinger, but you can also use the forefinger and the middle finger, etc. It is important not to store the picture in mind but the quantity, because e.g. if you are in Germany „two" is represented by the thumb and the forefinger, in Turkey you use the forefinger and the middle finger. The important thing is the quantity of shown fingers. At the same time, this opens the possibility of different left-right-hand-decompositions like 7=5+2 or 7=4+3 (s. below).

Especially the decomposition in tens helps to understand the decimal number system. Another task that we offer to the children is to present higher quantities, e.g. 43. How many children do we need to present 43 all at once? How many children have to lay down all 10 fingers? How many fingers does the fifth child have to lay on the table? Another example may be "13" that is presented by one person (1 tens = 10 ones) and a second person with 3 fingers so we have tens and ones.

3. AT-based Analysis of Internalization and Externalization Between Subject and Artefact

Before we can describe the process of internalization and externalization between subject and artefact we must present an example of a multi-touch learning environment and describe the technical implementation in more detail.

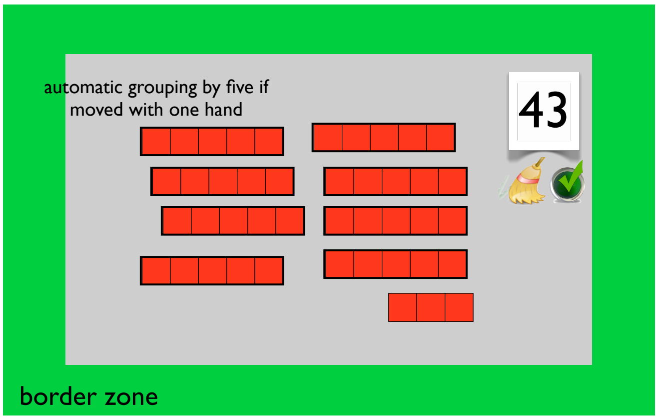

Fig. 6: Example learning environment

In the learning environment (Fig. 6) we ask students to place a certain number of (virtual) tokens on the table as fast as possible. This number is within the range of up to one hundred tokens. We expect students that have a fully developed number concept to use the structure of 10's to quickly reach a number such as 43 by placing four times 10 tokens and then the remaining 3.

Placing tokens is done by moving one or more fingers from a specially designated "border zone" of the table into the center of the table. If students use a full hand the five tokens associated to the fingers are grouped into a bar of five. These bars of five can easily be recognized and used for keeping the process structure (using a full hand) in a visual form. Students can use this to make use of their number concept for easier (and faster!) placing of the correct number of tokens. The learning environment can support the students by allowing for a restructuring in fives and tens automatically or on demand. It remains a research question whether students can develop their number sense better if they are offered such structuring aids. The environment can be used either for diagnosis of the development of the number concept of the students or, in a second step, it could be used to enhance their concept using a supplantation approach with automatic structuring aids as described.

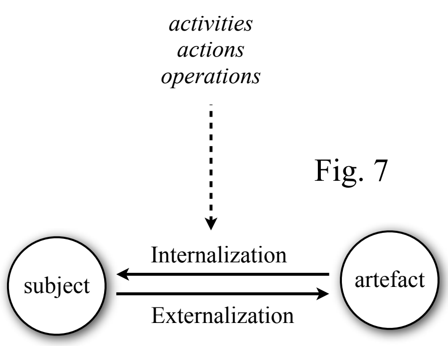

Fig. 7

In both cases it is helpful to view the work of the students at the table in the light of AT (Kuuti p. 30): The *activity* in question is solving the task to place 43 (or any other number) of tokens on the table. Students will work on several such activities during a session. As we ask them to solve as many number placing tasks as possible during a certain time frame they must work goal oriented. In the framework of Activity Theory we recognize a clear orientation towards the object: Students with a better understanding of numbers and their structure, that is with a fully developed number concept, can work on the activities much faster than students without such a competence. The activity – to transform the symbolic representation of a number into a cardinal-iconic representation – is created by our desire to understand the development of the students' number concept.

Each activity is divided into several *actions* of the student. Here, an action is the movement of one or several tokens from the border zone onto the table. Each of these actions consists technically of several touch-drag-release sequences on the table. Every single touch-drag-release sequence corresponds to an *operation*. Depending on wether these operations are carried out simultaneously or not, it can be judged through the programming of the artefact whether the student uses the "power of five" by himself or not. When the operations are both spatially and temporarily local – that is, they are taking carried out at almost the same time and place – the artefact will amplify the structuring approach of the student by creating a bar of five instead of five separate tokens.

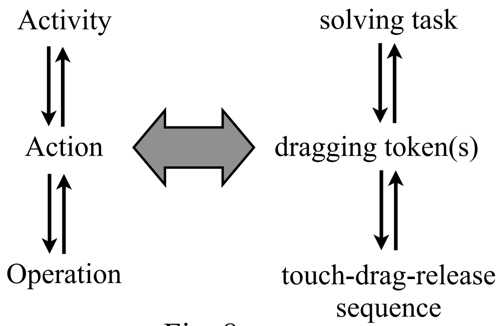

Fig. 8

From the students' perspective the actions will be collapsed into operations if they work within the environment, as they will not consciously move single fingers but full hands (or several fingers if they place less than five tokens). Still, from a technical point of view and for the design of the learning environment it is helpful to stay with the granularity of operations, as they are the key to implementing the rules for the multi-touch artefact. Collapsing the actions is only useful if we can rely on libraries for creating multi-touch environments that integrate the programming for operations and actions in a math-education-based manner. While this calls for creating programming environments that support maths education concepts, we do not know of any such environment that is freely available yet.[1] This is different for mathematics – there are special environments that encapsulate mathematical theory and offer this to the designer of activities (Richter-Gebert & Kortenkamp, 2010). This is the reason why we chose Cinderella for implementing our prototypes, as this can handle multi-touch input and supports mathematics properly. In this software, multi-touch operations are supported via a *touch-locality* mechanism: For each finger (or touch-drag-release sequence) a separate context is created that can be handled individually. For the technical details we refer to (Richter-Gebert & Kortenkamp, 2011) as well as the implementation overview in (Ladel & Kortenkamp, 2011).
Students can realize each activity using different actions. We suppose that the choice of these actions depends on the model(s) the children use for representations of quantities. A student using the power-of-five approach already (say, by placing 5+5, 5+5, 5+5, 5+5, 3 to create 43) will benefit from creating tokens with one or two hands at the same time instead of creating them one-by-one or creating them in varying quantities (say, by placing 4, 7, 7, 6, 8, 9, 2 tokens to create 43). These two approaches can be differentiated by the artefact by combining the operations into actions and classifying each of them.

4. Instrumental genesis within ACAT

The internalization and externalization between student and artefact that constitutes of the activities, actions, and operations (A-A-O) as illustrated in the preceding section captures the instrumentation and instrumentalization of the learners. By transforming the tokens that mediate the number concepts (which are the underlying object) the students both can express their current understanding and their mental representation is influenced and evolves through this process. Therefor, the A-A-O perspective supports the instrumental genesis (Artigue), see Fig. 9. We do expect an observable change of the students' concepts, and we observe it by comparing how the actions and operations change over the time while working on activities.
Thus, we can take a process-oriented viewpoint on the competence of the children with respect to their number and operations concepts. Instead of assessing the outcome of their actions (*are they*

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[1] The Cabri Elem Creator software by Cabrilog is a very promising environment for creating maths education aware activities. However, it does not support multi-touch yet.

able to put 43 tokens on the table?) we assess the process (*how do they put 43 tokens on the table?*). In traditional environments we can only indirectly understand how developed a certain competence is by looking at the product. In particular, standard errors can be identified that point out certain misconceptions or under-developed competences. However, for such simple activities as putting a certain number of tokens on a table the explanatory power of wrong (or right) results is marginal. We can infer from the additional data from the actions and operations that occur during the activity whether the students are on certain level of competence, or better: what stage in the instrumental genesis has been reached.
Again, the limiting aspect of a virtual manipulative is (positively) influencing the learning process. Without the limitations through the environment students could do anything with the tokens, which we want to prevent. For example, students cannot create stacks of tokens, but they have to place them next to each other. This restriction together with the automatic structuring of bars of five helps them to use the grouping aspect for subitizing quantities. Bundling quantities by building stacks is a good approach in other contexts, but it does not help in visualizing quantities mentally.

5. Instrumental Orchestration

As a final aspect we ask for the location and role of instrumental orchestration with the ACAT framework. While instrumental genesis is located in the interaction between subject and artefact, instrumental orchestration is happening in the lower left triangle of ACAT.
In first experiments of our group we could confirm the results of Harris that in the multi-touch environment students communicate task-specific instead of role-taking specific (Dohrmann, 2010), confirming the results of (Harris, 2009). The teacher is unburdened from micro-organizing as the students can work with equal rights and at the same time. The role of the teacher changes here. In fact, we need the teacher for supervising the work of the students with respect to mathematics education aspects. As the multi-touch table does not and cannot observe how students work with their fingers before they touch it, the teacher should do so. If a student counts fingers in the air before placing them on the table, this is not recognizable for the artefact as a counting approach. The teacher has then the option to intervene and either encourage the students to work differently or help them one-to-one.

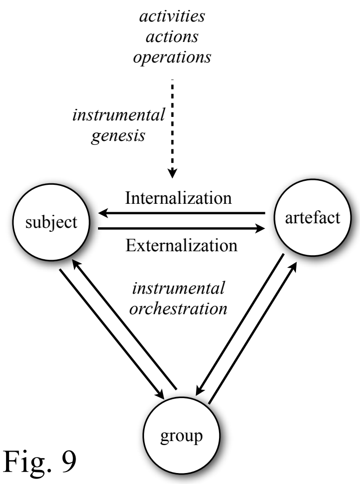

Fig. 9

6. Conclusion

Multi-Touch technology is a complex technological solution that involves numerous changes compared to traditional media. This affects both the interaction of the students with the artefacts (usually via virtual manipulatives) and the interaction of the artefacts with the objects (that is the mathematical content to teach). Also the orchestration of such instruments is of particular importance, as multi-touch-environments bear the opportunity of collaborative learning. Through the lens of a variant of Activity Theory that places the artefact in the center of attention (ACAT) we can locate the various areas of didactic and pedagogic design that have to be taken into account.
The assessment of learner’s progression can move from a product-centric to a process-centric perspective. Through computer-based analysis of operations as the most granular events we are able to identify typical actions for the different competence levels of students working with numbers and number operations.

Bibliography

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. International Journal of Computers for Mathematical Learning, 7:245-274.

Bottino, R. & Chiappini, G. Using Activity Theory to study the relationship between technology and the learning environment in the arithmetic domain. In English, L. & Bussi, M. (eds.): *Handbook of international research in mathematics education*

Brissiaud, R. (1992). A Toll for Number Construction: Finger Symbol Sets. In J. Bidaud, C. Meljac & J.-P. Fischer (eds.). Pathways to number. Children's Developing Numerical Abilities. New Jersey. Lawrence Erlbaum Associates.

Dohrmann, Chr. (2010). Über die didaktische Konzeption interaktiver Geometriesoftware angesichts substantieller Weiterentwicklungen von Mensch-Maschine Schnittstellen. Diplomarbeit. PH Schwäbisch Gmünd.

Drijvers, P., Doorman, M., Boon, P. & van Gisbergen, S. (2009) Instrumental orchestration - theory and practice. In V. Durant-Guerrier, S. Soury-Lavergne, & F. Azarello (eds.). Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education.

Engeström, Y. (1987). Learning by expanding: An activity-theoretical approach to developmental research. Helsinki: Orienta-Konsultit.

Engeström, Y. (1991). Activity Theory and Individual and Social Transformation. Activity Theory, vol. 7, no. 8. pp. 6-17.

Gelman, R. & Gallistel, C.R. (1987). The child's understanding of number. Cambridge, MA: Harvard University press

Harris, A. u.a. (2009). Around the Table: Are Multi-Touch Surfaces Better Than Single-Touch for Children's Collaborative Interactions?

Kuuti, K. (1996). Activity theory as a potential framework for human-computer interaction research. In Nardi, B.A. (ed.). *Context and consciousness: activity theory and human-computer interaction.* MIT Press, Cambridge, MA.

Mayer, R. (ed.) (2005). The Cambridge Handbook of Multimedia Learning. Cambridge University Press. New York.

Krauthausen, G. (1995). Die „Kraft der Fünf" und das denkende Rechnen. In Müller, G. & Wittmann, E.Ch. (eds.) Mit Kindern rechnen. Arbeitskreis Grundschule - Der Grundschulverband - e.V. Frankfurt am Main.

Kuuti, K. Activity Theory as a Potential Framework for Human-Computer Interaction Research. In Nardi, B. Context and consciousness: activity theory and human-computer interaction

Ladel, S. (2009). Multiple externe Repräsentationen (MERs) und deren Verknüpfung durch Computereinsatz. Zur Bedeutung für das Mathematiklernen im Anfangsunterricht. Didaktik in Forschung und Praxis, Bd. 48. Verlag Dr. Kovacs, Hamburg.

Ladel, S. (2011). Multiplex-R: Zum Wechsel zwischen verschiedenen Darstellungsformen von Zahlen und Operationen bei 5- bis 8-jährigen Kindern. In: Beiträge zum Mathematikunterricht. Münster: WTM-Verlag.

Ladel, S. & Kortenkamp, U. (2009). Virtuell-enaktives Arbeiten mit der „Kraft der Fünf". In: MNUPrimar 3/2009

Ladel, S. and Kortenkamp, U. (2011). Implementation of a multi-touch-environment supporting finger symbol sets. In *Proceedings of CERME 7, Rzeszow*.

Resnick, L.B., Bill, V., Lesgold, S. & Leer, M. Thinking in arithmetic class. In B. Means, C. Chelemer & M.S. Knapp (ed.), Teaching advanced skills to at-risk students: Views from research and practice (pp. 27-53). San Francisco: Jossey-Bass.

Richter-Gebert, J. and Kortenkamp, U. (2010). The power of scripting: DGS meets programming. *Acta Didactica Napocensia*, 3(2):67–78.

Richter-Gebert, J. and Kortenkamp, U. (2006). *Cinderella.2 (Computer Software). See http://cinderella.de*

Richter-Gebert, J. and Kortenkamp, U. H. (2011). *The Cinderella.2 Manual – Working with the Interactive Geometry Software*. Springer-Verlag.