Early maths with multi-touch
–
an activity-theoretic approach

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In this article we discuss the use of ICT, especially of multi-touch technology, to survey and to enhance the development of children's concepts of numbers. As a basis for the design and analysis of this research project we refer to the Artefact-Centric Activity Theory (ACAT), that is based on Engeström's structure of human activity systems. Adopting this theory, we focus on the different processes of internalization and externalization mediated by the artefact. The children externalize their concept of numbers through touching a multi-touch screen with their fingers and thus producing tokens. The object, that is a well-developed concept of numbers, determines the design of the multi-touch surface, that is also determined by general rules. The visualization of the multi-touch table contemporaneously gives feedback to the children that leads to an internalization. Hence the design of the user interface plays an important role. Also the partner as well as the (nursery) teacher and the kind of questions, that have influence on the child's externalization.

1. Introduction

In Germany, the use of ICT in kindergarten (as well as in primary school) is still seen very critical. One crucial point is that there is a lack of analyzing the processes happening while the children work with ICT and hence there only exist few didactical concepts of how to use it and how to design it. In our research activity we have a bifocal perspective on a meaningful use of technology in learning and teaching mathematics: We try to combine good design and usability of ICT (1) with the point of view from mathematics education (2). To support our analysis of the activities of the children with the focus on ICT we developed the Artefact-Centric Activity Theory (ACAT) (Ladel & Kortenkamp 2012). Using this theoretical framework it is possible to point out the different processes of internalization and externalization that take place while learning with ICT and to focus on the mediating role of the artefact. Didactical principles as well as design principles describe rules that have to be followed in the development of an ICT-based learning environment. Beyond that we are able to clear up the role of the nursery teachers.

In the project described below, from the mathematical perspective, we aim at a well-developed part-whole concept, because this provides the basis for several fundamental mathematical principles: commutativity and associativity of addition, or the complementarity of addition and subtraction (cf. Resnick et al. 1991, p. 32). It is also important to understand the decimal number system because this uses a decimal part-whole concept. Before the child progresses to a part-whole concept it develops an ordinal and a cardinal concept of numbers, that live in parallel to each other (cf. Dornheim 2008, p. 86). All three concepts can be externalized using fingers, and we can observe these externalizations for example when the fingers are used to touch a multi-touch table (MTT). Fingers are a very famous medium to represent quantities and there are numerous ways to do this (cf. Brissiaud 1992, Ladel & Kortenkamp 2009). The extremes are the child showing the fingers *one by one* or showing the fingers *all at once*. The starting point of our development of multi-touch learning environments is to survey the way children work with their fingers. In a first step we will just observe and recognize the touches and the spatial and temporal distances caused by a child laying his fingers on the table surface.

Considering those things our research questions are:

- Can the collected data help to distinguish the three number concepts that children might have?
- How must a multi-touch environment be designed to collect the appropriate data?
- Can we support the development of the part-whole concept with such environments?

2. The ACAT Theoretical Framework

> *„Studying the changes that learning environments undergo when technology-based artefacts are introduced means analyzing how activity changes as consequence of tools' introduction and how this change is meaningful for the students and the teachers.“* (Bottino & Chiappini 2001, p. 841)

As mentioned above, we relate our analysis to Artefact-Centric Activity Theory (ACAT) (Fig. 1) that is based on Engeström's structure of human activity system (1987).

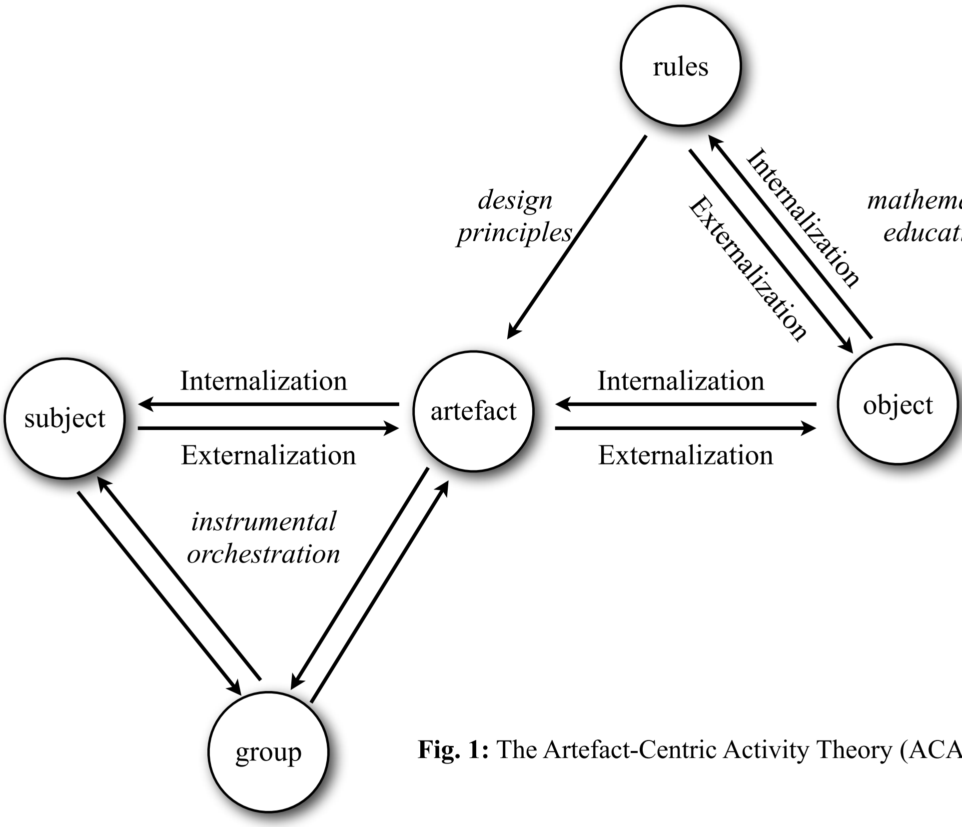


Fig. 1: The Artefact-Centric Activity Theory (ACAT)

In Activity Theory we consider the whole learning environment including the community (group) and the rules. The arrows show the resulting interrelations. An activity is a form of acting directed towards an object (cf. Bottino & Chiappini, 2001). The *object* in our work is the development of the part-whole concept with the *outcome* of an educated student (Bellamy, 1996). Instead of measuring the outcome we try to assess the processes that occur during the activity. ACAT gives us a framework to describe and analyze the interactions of children with the artefact to reach the object. The main axis of ACAT is built along the subject, the artefact and the object, where the artefact acts as a mediator between subject and object. We moved the artefact in the centre, because it constitutes the focus of our research.

The object of our learning environment is to support the development of the part-whole concept of the children through the use of ICT. The use of ICT in kindergarten in Germany is seen very controversial. This is mainly due to the existing software that rarely respects mathematics-specific didactical principles and the way young children learn mathematics (Fig. 2). Another point is that young children do have problems manipulating the computer, for example caused by the indirect manipulation of objects via keyboard or mouse. It is difficult for them to coordinate their eyes and what they see on the screen with the movements of the mouse (hand-eye-coordination). Furthermore, the scale changes, the distance moving the mouse doesn't correspond to the distance on the screen. Even the question what to do if the mouse arrives on the border of the table or mouse pad is challenging in the beginning.

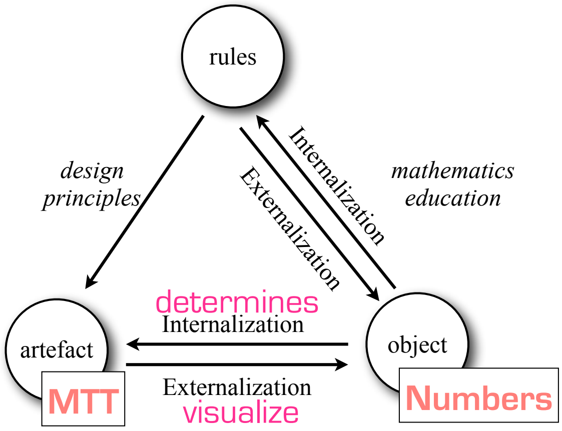


Fig. 2: object - rules - artefact

In the last years human-computer-interfaces have evolved to direct manipulation using touch-sensitive interfaces. This technology enables children to work directly with virtual manipulatives: interactive, visual representations of dynamic objects that provide opportunities for constructing mathematical knowledge (cf. Moyer, Bolyard & Spikell 2002). Furthermore, it is possible to create environments with large screens, like MTTs, that encourage collaborative learning and communication of the children.

Multi-touch technology opens a new field for virtual manipulatives as it enables children to work in an even more natural way. It is now possible to touch the screen with *several* fingers at once instead of just one finger at a time. This meets the way young children learn. Multi-touch technology is a matter of particular interest for our area of part-whole number concepts: Children are able to represent a quantity by touching the screen with many fingers, while "traditional" computer environments must use ordinal concepts or symbols just for entering quantities. We suppose that the way a child uses this input can give us more information about the concepts of numbers it uses.

At about the age of 2 to 5 years children develop an ordinal concept of numbers, where numbers describe positions in an ordered row (cf. Fuson 1992, Dornheim 2008). Coming along with that is the knowledge which number is the successor or predecessor and where a number is located on the number line. Objects can now be brought into a relative position to each other. Parallel to this (ca. 3 to 6 years), children acquire also a cardinal concept of numbers. They understand that number words not only can be used to count, but also to name a quantity. The last number counted corresponds to the cardinality of a set (cf. Gelmann & Gallistel 1978). The part-whole concept of numbers is based on this processes. Several parts are composed to a whole, e.g. the parts 3 and 5 are composed to the whole of 8. A special part is the decimal part-whole concept that combines the part-whole concept with the decimal structure of our number system (Fig. 3) (cf. Ladel & Kortenkamp 2011, Ladel 2011).

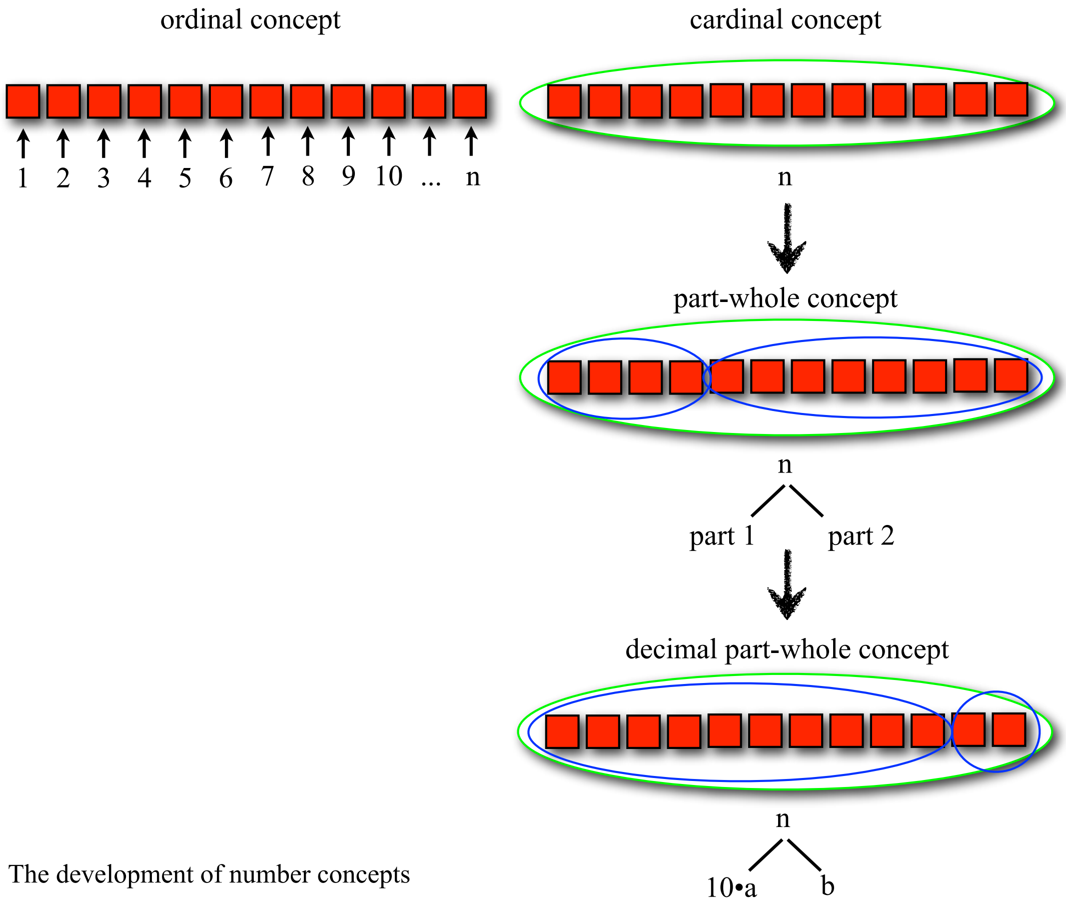


Fig. 3: The development of number concepts

The *subject*, here the child, externalizes its concept of numbers by putting his fingers on the MTT. According to the different concepts of numbers we can distinguish the following ways: counting, one finger by one (ordinal concept), all fingers at once (cardinal concept), all fingers at once but separated in different parts (part-whole concept), all fingers at once, separated in tens and ones (decimal part-whole concept). The MTT as the *artefact* externalizes by a suitable representation and visualizing the actions of the the children as tokens. The *object* is not only what the children should learn, but it also determines the way the MTT software has to be programmed. Therefore, we have to follow certain rules, e.g. mathematic-didactical rules and multimedia design principles (cf. Ladel 2009). Finally, the experiences the children have while working with the MTT environment can lead to the desired internalization.

In this theoretic framework, the role of the nursery teacher is to supervise the work of the children, and to take care of aspects that cannot be handled by technology. For example, all actions done with the fingers *before* touching the screen cannot be recorded by the computer. The teacher can observe these actions and these observations can be included in the analysis.

3. Analysis

We developed a firs environment where users can produce tokens by touching the green border of the screen. Tokens that are moved into the center of the table will remain. If a finger is released while still on the green area, the token will vanish again. In the following section we will analyze some examples that show how children worked within this environment.

Task: „*Please put x tokens on the table.*“

1	Interviewer	*A., please put six tokens on the table.*
2	A.	*Ok. First the first finger* (A. uses the thumb to move one token on the table)
3		*then the second finger* (A. uses the index finger to move another token on the table)
4		*then the third finger* (A. uses the index finger again and moves another token on the table)
5		*ah* (she shakes her head and wants to take the token back with her middle finger, but the table doesn't react)
6		*I just take the forth* (A. takes her ring finger and wants to move one token on the table, but the token on the table doesn't move due to technical reasons)
7		*then I just take this one* (A. uses the index finger to move a fourth token on the table.)
8		*and this one* (A. looks at her fingers and uses the index finger again to move one token on the table.).
9		*One, two, three, four, five. One more* (A. uses the index finger again to move the sixth token on the table)
10		*and six. One, two, three, four, five, six.* (Counting the tokens)

A. began by moving tokens one by one in the center of the table. But she did not rely on counting "one, two, three, ..." but she connected the tokens 1-to-1 to the fingers and used the ordinal numbers "the first, the second, ..." This stands to reason that the ordinal concept of numbers dominates, where the number word "one" corresponds to the first finger, "two" to the second finger, etc. But in line 4 we see an *adjustment event*: A. made a mistake by using the 'wrong' finger. She used her index finger, which is „the second“ for her, to produce „the third“ token. She recognized her 'fault' and wanted to correct it by removing the token with her middle finger.

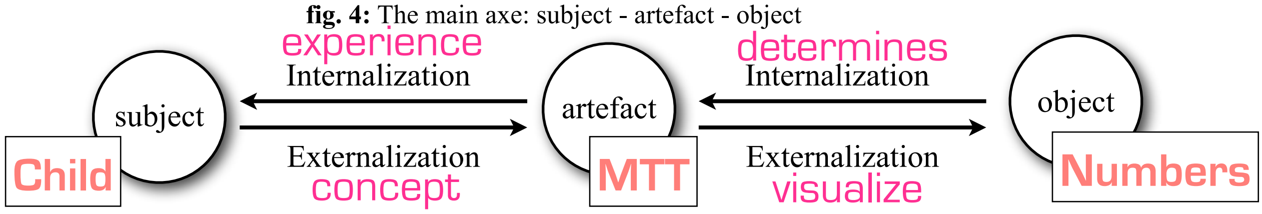
A. externalized her concept of numbers via putting her fingers on the MTT and the MTT visualized the fingers through the tokens. At this stage we don't know, if A. really identifies only the sixth finger or the whole quantity with the number six. But then, the visualization through the MTT didn't work well, which leads to the situation that A. had to change the concept she uses. She either knows or experienced that it doesn't matter with which finger the token is moved and accepted to let the third token remain on the table.

Next she switched back to her ordinal concept and tried to move a token with her ring finger. At this point there is a second adjustment event shown by the change of concept, this time caused by a technological fault of the MTT that caused A. to use her index finger again instead of her ring finger. When she wanted to place the fifth token she first thought about which finger to take and then decides to use again the index finger, assumedly because her experience now tells her that the table reacts best with this finger and it is easier as well for the fine motor skills. Most importantly, she already learned that it does not matter if she uses the 'wrong' finger. The unresponsive table caused a transition of concept.

In the end A. proved her work by counting all tokens on the table. She did not only correspond the sixth token with the number six but the whole quantity of all tokens on the table. This means that she already connected the ordinal with the cardinal concept of numbers.

This example shows very well how the artefact can influence the way children work and also evoke changes of concepts (Fig. 4). If the MTT would have worked well and recognized A.s fingers, there would never have been need to change the concepts.[1] We could observe some children that payed attention to the fact which finger they take to produce tokens, but than changed their concept and took any finger. The experience with the MTT leaded to the fact, that the children were able to abstract and knew, that it doesn't matter, with which finger a token is produced.

fig. 4: The main axe: subject - artefact - object



Furthermore we could observe that the young children did have some problems producing tokens. E.g. it wasn't easy for them to move the fingers on the screen and therewith the tokens away from the body. The children couldn't see the tokens because they were covered by their hands (s. Dohrmann 2010). Also the multi-touch technology had problems with the recognition of the fingers, because the young children touched the screen too weakly. Hence the table didn't visualize all the fingers as tokens. This led to the fact, that children preferred using the one-by-one-method (counting) instead of all-at-once and hence a simultaneous or rather quasi-simultaneous representation. So the experience the children made with the artefact changed their behaviour in a way we didn't want it at all. We had to change the design of the MTT.

In the second experiment the environment was changed. The green area was moved into the center of the table and the children could create tokens by "pulling" them out of that area to the border of the table. We could observe that this changed user interface encouraged more children to create several tokens at once.

We also recognized that the formulation of the task was of importance. If the task was formulated as. *„Please put x tokens on the table,"* then a lot of children created them one by one. This could be caused by the usual interaction with real tokens. For these it is difficult to grab a specific number, because each finger would have to be placed exactly on top of a token. With different material, for example the abacus, this is different – here, children can "take"

[1] We do not dare to claim that technology causes learning due to it insufficiencies, we just point out how complex the interactive process of working with an electronic learning environment can be, both in the intended and the unintended way.

beads in groups of five, for example, in a single move. In single-touch computer-based learning environments the virtual manipulatives usually have to be taken one by one as well (for exceptions cf. Ladel & Kortenkamp 2009). In the multi-touch-environment it is possible to create several tokens at once without the need to place the fingers exactly, quantities can be created quasi-simultaneously, but the children (or other users) are not aware of this option. So we changed the formulation of the task into:

<u>Task: *„Please put x tokens on the table, all at once.“*</u>

The children reacted in various ways. Some children instantly changed to another concept and used the corresponding number of fingers on the table, demonstrating the importance of the formulation of the task. It also happened that a child was not familiar with the term „all at once“ (German: *gleichzeitig*). One boy asked *„What does that mean, all at once?“*

To analyze the processes we also implemented automatic recording of the children’s touch actions. We are thus able to analyze the externalization process of the students also using the collected data. The recorded data is demonstrated in the following figure, where you can see when and how long a finger touched the screen. On the left the screen was touched with three fingers at the same time. On the right we can see that five fingers touching one by one.

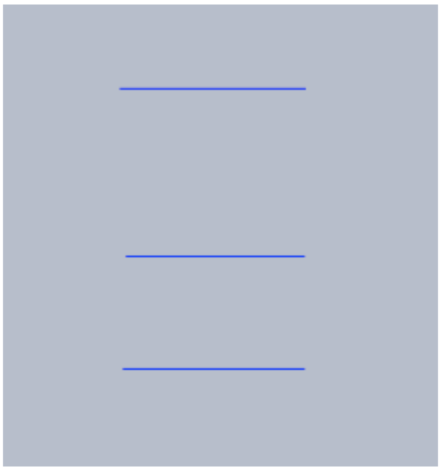
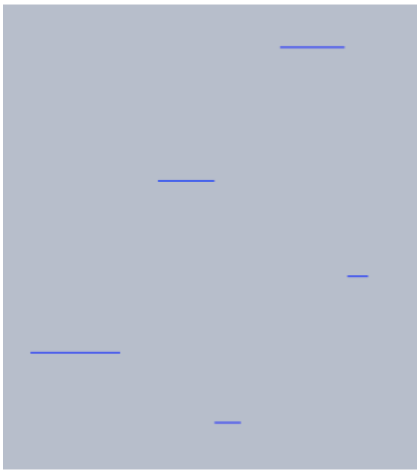
three fingers all at once	five fingers one by one
	

Fig. 5: Log-data of finger-touches

This data and its visualization may help to analyze the applied concepts a posteriori. Still, we have to be aware of the fact that the fingers are already a mediating artefact on their own. Using them creates a first externalization.

In our experiments we could observe children changing their concepts when they were first representing numbers with their fingers and then representing the same numbers with their fingers on the MTT: Some children first counted their fingers one by one and then put them all at once on the table. Other children did the opposite, showing fingers all at once when asked for a certain number, and working one-by-one on the MTT.

From this, we deduce the need for a nursery teacher to work with the children and observing them, as it is impossible to capture the full picture with technology alone..

Another influencing factor is the partner of the subject. When a child sees in what way the partner put its fingers on the table, it will try to match this usually, unless the own concept is stable enough to persist. The following excerpt demonstrates this.

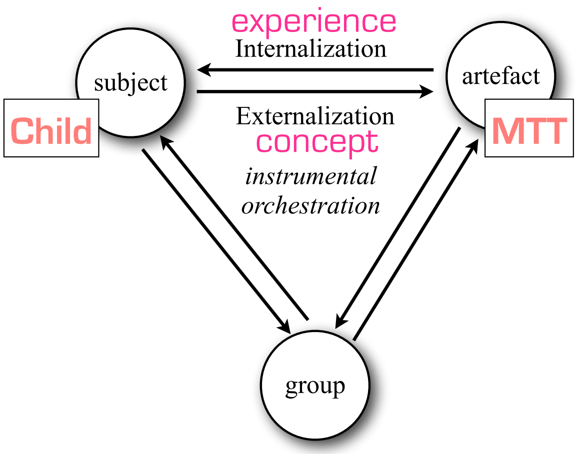


Fig. 6: subject - artefact - group

1	Interviewer	*A., please put five tokens on the table, but this time all at once.*
2	A.	(A. put all five fingers of her right hand on the screen and moves the tokens on the table.)
3	Interviewer	*V., could you please put four tokens on the table, all at once?*
4	A.	*Like this.* (A. put four fingers all at once on the table and lifts her hand again.)
5	V.	(V. uses her index finger and moves tokens one by one on the table.)
6	A.	*No, like this.* (A. shows V. four fingers all at once.)
7	V.	(V. looks at A.s fingers and continuous to move tokens one by one)

A. moved five tokens at the same time and V. watched her doing it. But as it was her turn she didn't do it the same way but moved them one by one, even when A. insisted that she should use four fingers and showed it to her again. V. couldn't change her concept but persisted moving one by one.

A theoretical explanation can be found in Vygotskys Zone of proximal development (ZPD). We must respect and observe, whether a child is able to change its concept with help or not. In the example above, A. was able to use the cardinal concept and use several fingers at the same time on the screen. V. wasn't able to switch to this concept, even when A. showed her how to do it. The new concept was not yet within the ZPD of V. In our setup we are able to distinguish children that can reach certain concepts from those who are unable to do this yet even with the help of their peers.

To examine the children's part-whole concept, a given task was to present a quantity of tokens „together", and another one to add tokens to a whole.

<u>Task: *"Can you put x tokens on the table together?" (German: Könnt ihr x Plättchen gemeinsam auf den Tisch legen?)*</u>

Many children did not understand the expression “together” in the intended way. Instead of placing a certain number of tokens on the table, each of them placed this number of tokens, ending up with twice the number of tokens desired. Even asking them how many tokens they do have together revealed only the correct number, that is, half of what was on the table, as each child counted his own produced tokens. Assumably the word “together” needs to be made clear in advance.

Nevertheless, we could observe a proper part-whole concept with some children. During the first experiments we could observe three ways of decomposition:

Halving:

The children decomposd into halves, e.g. *„Six, that is twice three.“* This way to decompose is used for even numbers between 5 and 10. With quantities larger than 10 the kind of decomposition changed.

Decimal part-whole concept:

Quantities higher than ten were decomposed into tens and ones.

P. did it as follows:

1	Interviewer	*Can you put twelve tokens on the table, together?*
2	P.	*One, two, three, four, five, six, seven, eight, nine, ten.* (P. counts and tips his fingers.)
3		*Eleven, twelve.* (P. continues counting the fingers of his partner.)
4		*S. you have to put two.*
5	Interviewer	*So how do you decompose the twelve?*
6	P.	*I make ten, and S. makes two.*
7	Interviewer	*And can you also move 14 tokens on the table, together?*
8	P.	*One, two, three, four, five, six, seven, eight, nine, ten.* (P. counts and tips his fingers again.)
9		*I make ten, and S. makes four.*

P. didn’t know or maybe wasn’t sure how many fingers he has on two hand and hence had to count them twice. The first time – representing twelve – he just went on counting up to twelve, but the second time – representing fourteen – he immediately knew that S. has to produce four tokens if he produces ten. He was then able to decompose 14 into ten and four.

‚Power of five‘:

A third way we observed was the use of the ‘power of five’ (cf. Krauthausen 1995. In German “Kraft der Fünf” is unambiguous, as *Kraft* does not mean power in the sense of the arithmetic operation, but only power in the sense of force). The task was to put twelve tokens on the table together. Before moving the tokens on the table, E. said: *„I make five and she makes five then and then I make two.“* E. did already have a sophisticated part-whole concept that she could use in the sense of addition. But she did not use the decimal part-whole concept and decomposed in ten and two but took the 'power of five' approach which corresponds to using one hand as bundling unit.

4. Conclusion

Children have different concepts of numbers, and make use of various concepts depending on the situation. It seems to be of deep importance in which context children are asked to make use of their concepts. In our experiments we could reveal various interactions between the mediating artefact and both the use of concepts and the evolvement of concepts. The ACAT framework helps us to observe and to analyze the processes and interrelations that take place.

Right now, we are at the very beginning of our research in multi-touch-enabled learning environments, where we only observe the complex effects of using a different and unusual, though seemingly natural user interface to virtual manipulatives. In further work we have to deepen this analysis in order to create helpful learning environments. A possible direction is, for example, the automatic arrangement of tokens creating a semi-structured virtual environment.

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