FINGER-SYMBOL-SETS AND MULTI-TOUCH FOR A BETTER UNDERSTANDING OF NUMBERS AND OPERATIONS

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Basic concepts of numbers and operations are fundamental for mathematical learning. Suitable materials for developing such basic concepts are hands and fingers. Among other things, this is because of their natural structure of 5 and 10. To support the development of concepts and the process of internalization a linking between different forms of representations by the computer can be helpful. To benefit of both, the advantages of the hands and fingers and the automatically linking, we suggest using multi-touch-technology, i.e. computer input devices that are able to recognize several touch gestures at the same time. Here, children can present numbers with their fingers that produce virtual objects. These objects can be automatically linked with the symbolic form of representation.

THE ORDINAL AND CARDINAL CONCEPT OF NUMBERS AND OPERATIONS

"How many things are there?" – For parents as well as for mathematicians, this is a common question to pose, if a child already has knowledge about numbers. For the child, this question is almost always the initiation to start counting verbally by saying the number words in a row (Fuson, 1988). The fundamental principles needed for answering the question are a) the one-one-principle that relates every single object to exactly one numeral (Gelmann & Gallistel, 1978), b) the stable-order-principle prescribing the correct order of numbers (Fig. 1, left), and c) the last-word-rule that assigns the last said numeral not the last counted object, but to the quantity as a whole (Fig. 1, right).

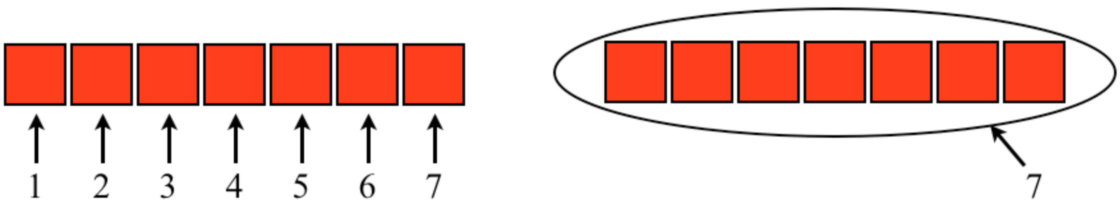

Figure 1: ordinal (left) and cardinal (right) concept of numbers

Here, the change from the ordinal concept of numbers, where the numeral is part of the numeral row, to the cardinal concept of numbers, where the numeral identifies a quantity, is necessary. It is not necessary to count a quantity in order to know it, that is, the ordinal concept is not a necessity for the cardinal concept. Resnick (1991) distinguishes the development of mathematical knowledge by two components that are developed independently: *protoquantitative schemata* and the *mental number line*. To build-up a well-developed concept of numbers, these two threads of development have to be linked. For many children this is a critical problem. Fuson refers to the following example: a child counts a quantity of five cars. As an answer to the question *„How many?“* the child points to the last-counted car and says: *„This is the five cars.“* (Fuson, 1992 p. 63).

Children who do not have a proper linking between the two concepts can misinterpret addition and subtraction as a demand to count forwards or backwards (fig. 2 left). As long as the children calculate with numbers smaller than 20 they can apply this strategy successfully. But, for instance, when they want to add 55 to 27 and begin to count *„28, 29, 30, 31, ...“* there is no chance to come easily and quickly to the correct result. Thus, it is important that children acquire a part-whole schema of numbers as a foundation for addition and subtraction (fig. 2 right).

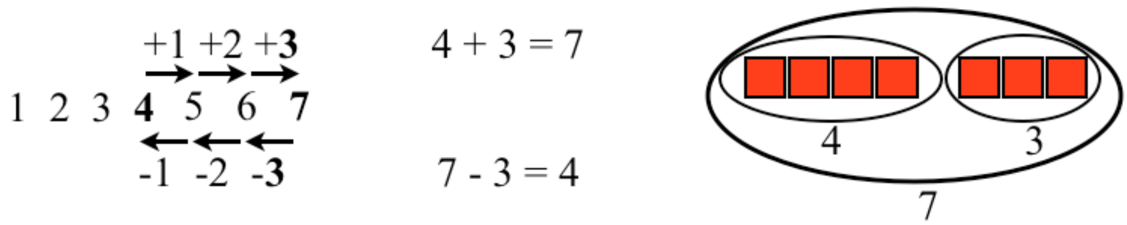

Figure 2: Addition and subtraction with the ordinal (left) and the cardinal (right) concept of numbers

„The protoquantitative part-whole schema is the foundation for later understanding of binary addition and subtraction and for several fundamental mathematical principles, such as the commutativity and associativity of addition and the complementarity of addition and subtraction. It also provides the framework for a concept of additive composition of number that underlies the place value system.“ (Resnick, 1991 p. 32).

For example when you want to add 6 and 8 with the use of the part-whole schema you can split and add in lots of ways (e.g. fig. 3).

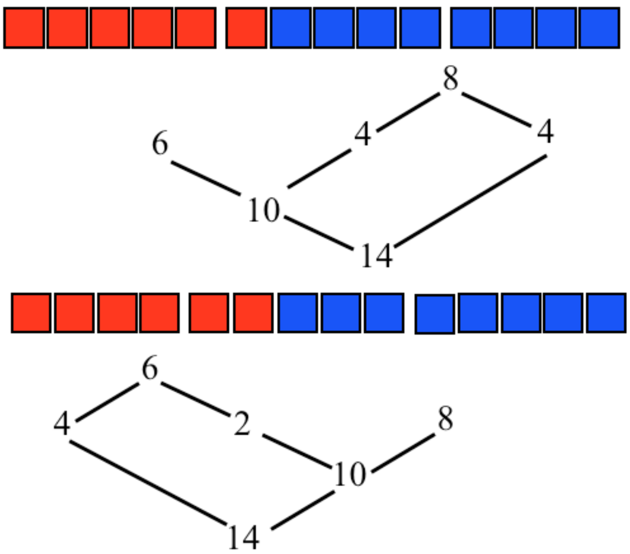

Figure 3: different ways to add with the part-whole-schema

Because of our decimal number system and with regard to the decimal analogy the secured understanding of the number range up to 10 is assigned a key position in the development of mathematical competences (Claus & Peter, 2005 p. 11). If you know all the possible decompositions of the numbers up to 10, you are able to add and subtract in bigger number ranges easily, e.g. if 7 + 8 = 7 + (3 + 5) = (7 + 3) + 5 = 15, than 67 + 8 = 67 + (3 + 5) = (67 + 3) + 5 = 75. The missing knowledge of and fluency in decompositions of the numbers up to 10 as much as the lack of capability to recall them quickly and effortlessly is the cause of many subsequent difficulties in mathematical learning.

FINGER SYMBOL SETS

Calculating with fingers has a very bad reputation in mathematics lessons, as it is usually seen as an indicator for counting. Most children do as they have learned from young days on and count objects by „Counting-Word Tagging to Number" (Brissiaud, 1992). According to the ordinal concept of numbers each finger is related to exactly one numeral. But if each finger is labelled by a number, counting children are encouraged to stay with their strategy and this consequently leads to further problems. To illustrate this we ask what happens if the sixth finger is buckled? The „name" of the last finger, that indicated the quantity, was „10" before, but now the finger has to be renamed into „9" (Fig. 4).

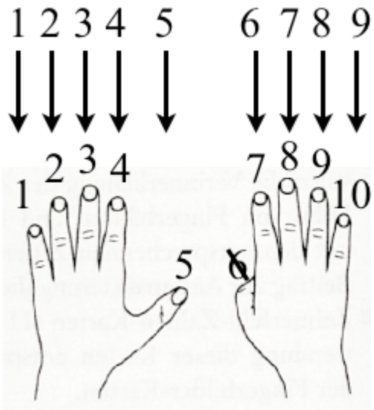

Figure 4: Order-irrelevance principle

This procedure can puzzle some children. Therefore it is important not to assign names the fingers – there is no "6-finger." The child has to know that it is irrelevant which fingers it uses to present a quantity. To present „3", the thumb, the index finger and the middle finger can be used as well as the little finger, the middle finger and the thumb, or any other combination of three fingers. As we point out below, the cognitive process behind this fact can be experienced and thus supported by the use of multi-touch-technology.

Amongst others, the advantages of fingers and hands are their permanent availability and their natural structure in *10* fingers per child with *5* fingers per hand. The 10 fingers qualify the hands to work out questions about the decimal number system, e.g. *„How many children do we need to see 30 fingers all at once?"* The 3 children stand for 3 tens. Just as well, the different decompositions of all numbers up to 10 can be presented with the hands. The „power of five" (Krauthausen, 1995) is due to the ability to instantaneously recognize quantities (subitizing) up to 4. Applying this to the hands, the shown quantity of the fingers of one hand can be conceived simultaneously and hence the fingers of both hands can be conceived quasi-simultaneously. Furthermore, one hand gets a special status because children tend to present numbers greater than five sequentially (Brissiaud, 1992 p. 61). For example, to present „7", they tend to use one full hand and then add two fingers of the other hand. In this way the decomposition of the numbers from 1 to 10 with the power of five can be worked out. But not only these, also all other decompositions are possible (Fig. 5) and can be conceived quasi-simulataneously.

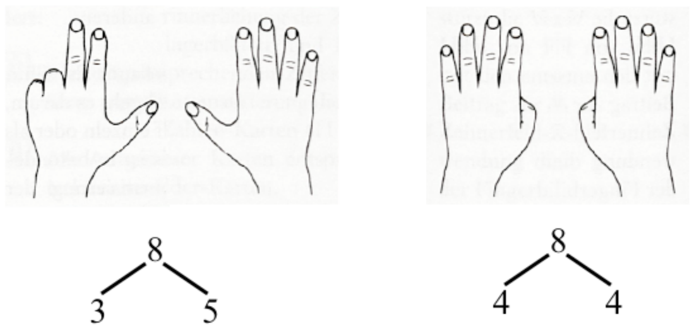

Figure 5: Decomposition of numbers with finger-symbol-sets

If the fingers are used like this, in sense of the part-whole schema, they are a qualified working material for a well-developed concept of numbers and operations (cf. Steinweg, 2009). Brissiaud (1992) coined the notion „From Finger Symbol Sets to Number":

„Certain children who were not exposed early to the use of finger symbol sets may become counters, whereas children who were encouraged to use finger symbol sets may preferentially choose finger strategies".

He could show that this way of gestured representation of quantities by some children is established early and in parallel to the development of the numerical row as an autonomous type of numerical representation.

The decompositions of all numbers up to 10 that were acquired in this way can now be utilised in the following process of mathematical learning of addition and subtraction (Fig. 6).

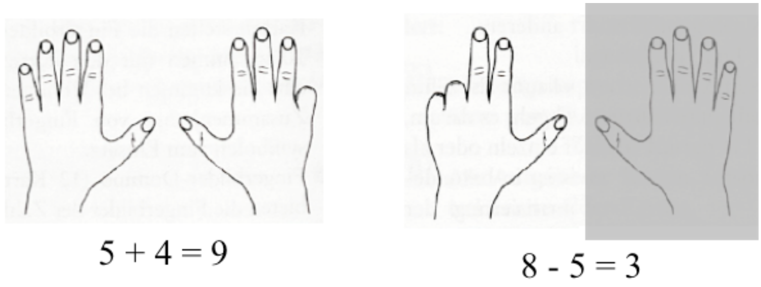

Figure 6: addition and subtraction with finger symbol sets

If children have a part-whole schema of numbers the transition to addition and subtraction is easy. It is just another way of nonverbal-symbolic representation of the fact that „two parts make a whole".

Further strategies like variation in the opposite or in the same direction can than be worked out easily: If one finger is buckled, than another finger must be stretched to keep the same quantity. To get the difference of two quantities, e.g. of 9 and 7, you can vary the numbers in the same direction. For example, a whole hand can be omitted, which corresponds to subtracting five from each quantity. It is evident that the difference of 9 and 7 is the as the difference between 4 and 2 (Fig. 7).

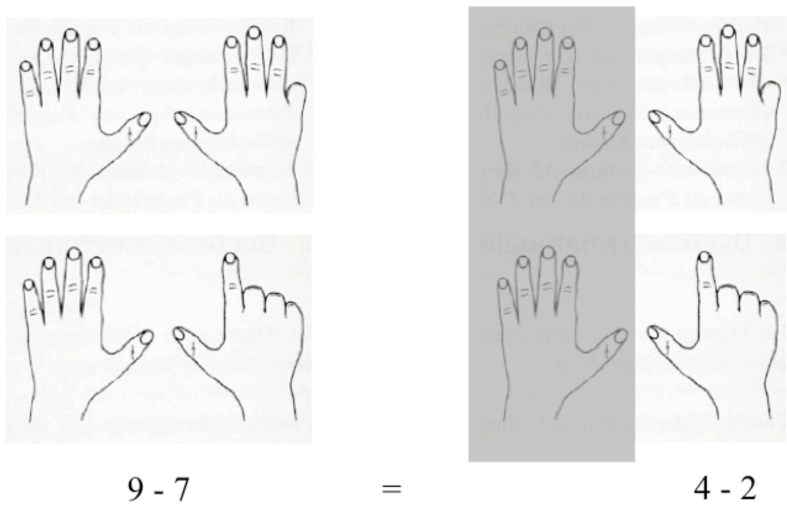

Figure 7: Strategy of variation in the same direction with finger symbol sets

Based on such strategies the decadic analogy can be build up.

It is important to pay attention to the fact that the children stretch their fingers *simultaneously* to represent quantities with them. If they show them one-by-one the positive effects of these strategies are lost and the children will still use counting for addition and subtraction.

As well as addition and subtraction, multiplication and division can be presented via hands and fingers. If 5 children all show 6 fingers, this represents five times six. The inverse operation is found when we start with 30 fingers that shall be represented by 5 children.

This introduction can only serve as a small insight into the possible representations of numbers and operations by hands and fingers and their usage in early arithmetic. It is the process of internalization that is of essential importance: How can the children benefit from the mathematical content of these representations and actions and use them in their mental processes?

THE PROCESS OF INTERNALIZATION SUPPORTED BY THE USE OF MULTI-TOUCH-TECHNOLOGY

According to Aebli, the process of early mathematical learning follows four stages, independent of the arithmetical subject (Grissemann & Weber, 2000; Aebli, 1987). Coming from concrete manipulations with different objects (stage 1), the children have to abstract these manipulations and operations to pictorial representations (stage 2). Subsequently they pass over to symbols (stage 3) with the aim to automate their actions (stage 4). For us, stage 2 is of special importance, because there the process of internalization takes place. The child has to comprehend the manipulation of concrete objects as a representation of a quantitative structure and it has to capture the structure and the relations of the concrete manipulation in an intellectual activity (Gerster & Schultz, 2004 p. 47). Lorenz calls this process „focus of attention". To facilitate this process of focus and abstraction and to develop it, a dialog is essential (Lorenz, 1997): *„In talking about the working material and the relations between numbers and operations that it represents, the concepts in development of the learner are going to be clarified by verbalisation."* In this sense, Aebli (1987) suggests that the children should review their concrete manipulations and make forecasts about further actions. Doing this, they comment their own manipulations by iconic illustrations till they are able to reproduce the structures and relations of the manipulations in conceptions. To support this process Aebli (1987 p. 238) established the following rule:

„Every new, more symbolic representation of the operation must be linked as closely as possible with the precedent one."

As shown in Figs. 5 to 7, the enactive form of representation with finger symbol sets should be related to the nonverbal-symbolical form of representation (MER[1]) (Ainsworth, 1995; Mayer, 2005). But as studies show some of the children even don't link the different forms of representations when they are designed in form of MERs (Clements, 2002). For them, an automatic linking designed with the computer (MELRs[2]) can help them to experience the relations (Thompson, 1992; Clements, 2002; Ladel, 2009). This experience should be as natural and directly as possible. In this article we suggest to use multi-touch-technology for this experience, where the children can manipulate with their hands and fingers and an automatic linking with all other forms of representation can take place. In the remainder of this article we assume the availability of a multi-touch-enabled table. Such a table consists of a display surface connected to a computer and some tracking hardware that can recognize several touches on the display simultaneously and report them to the computer software. Similar technology with a different form factor is available in desktop monitors, tablet computers and devices like the Apple iPad, or mobile phones. With the availability of hardware as already imagined by Kay (1972) we now have to answer the question of the educational implications more than ever.

The basic underlying idea for all the activities sketched only briefly in the following is that the computer can track the children's actions on the table and give nonverbal-symbolic representations of either the current situation or the action that lead to it in form of a written protocol.

In a first scenario, the children represent numbers with their hands and fingers as described before. This enactive form of representation shall produce an iconic one on the display. The computer creates quadratic pads on the surface of the multi-touch-table. Already at this stage, the focus of attention of the children can be laid on the fact of bundling the 5 fingers of one hand to a bar of 5 and the 10 fingers of two hands to a bar of 10. Through the contact of the fingers with the multi-touch-interface there is not only a link between the enactive form of representation with other forms of representation but also between the tactile and the visual sense. While representing numbers enactively and thus iconically, there is an automatic link to a nonverbal-symbolic form of representation. This representation can be imagined like a paper tape or sales slip and serves as a kind of protocol for the manipulations the children do. Such a protocol can support the focus of attention and the numerical aspects of a task (Dörfler, 1986).

In this activity it is possible for children to experience that it is of no particular importance which fingers they use to present quantities. Using the thumb and the index finger or using the little finger and the ring finger both yield the number "2". At a table, it is also possible that the children work in teams: Two children can "share the work" to present two fingers if each touches the table with one finger. While this sounds funny for the number two, it is of great importance for partitions of larger numbers. Two partners can try to find all ways to partition numbers up to 20 into two numbers up to 10.

Working in teams or groups the children are also able to present numbers greater than 10, emphasizing the social aspects of learning. Because the protocol immediately reflects the actions of the children their focus of attention is on the mathematical content of their actions automatically, guiding them to abstraction.

We can also use the technology as a diagnostic tool for the number concepts of children. Recording the way the children touch the table with their fingers it is possible to measure the time intervals between the touches of each finger. We can judge whether the children are still counting to put a given number of fingers onto the table, or whether they work already with suitable finger symbol sets.

Just showing the number of fingers touching the surface can also help the children to experience the difference between the ordinal and the cardinal aspect of numbers.

The child can touch the table with its fingers in different sequences, different positions or with different fingers – in all cases the protocol just depends on the quantity of fingers.

It is also possible to support the four basic arithmetic operations and their basic concepts in such an environment. Regarding addition, students can develop the basic concept of a union by manipulating the virtual objects (pads) and arrange them close to each other. For example, if the child merges a group of 3 pads and a group of 5 pads the protocol will show the symbolic representation of this action as „3 + 5 = 8". Here the focus of attention lies on the fact that this action constitutes a basic concept of addition, together with its nonverbal-symbolic form of representation. In multi-touch-technology there is also the possibility to draw a circle around some pads with the effect that these pads are bundled (a so-called lasso-gesture). This again is a manipulation based on the basic concept of union. Another task in the realm of addition and subtraction may be that 3 pads are shown and the child should create so many pads that in the end there are 7 (3 + _ = 7).

The basic concept of balance can be represented as well. Children can create quantities, remove from them, manipulate them with their fingers, and see the consequences of the manipulation at the same time in the nonverbal-symbolic protocol. Likewise it is possible to give instructions in the nonverbal-symbolic form and to see the output in the iconic forms with the pads.

It is rather easy to imagine that addition and subtraction can be done in such an environment, and we have shown some ways how the action or the state can be linked to a nonverbal-symbolic representation. For multiplication and division it is advisable to take advantage of the time as another dimension. The temporal-successive idea of multiplication that can be traced back to a repeated addition is mapped to a repeated touch action of the same quantity of fingers several times. The protocol may then show, for four touches with five fingers, "5 + 5 + 5 + 5 = 20" as well as "4 • 5 = 20". Thus the children can see, that there are different ways to protocol their manipulation. If several children are working together they can take advantage of the spatial-simultaneous idea of multiplication, creating the same quantity by several children at the same time. For division, one example activity would be to move pads and build piles of the same amount to divide a given number of pads.

FORECAST

We are currently working on implementing the above scenarios using a multi-touch-table built at CERMAT. A first study that examines the critical point in translating numbers and operations from and in different forms of representation is going to take place in October 2010. At the same time we will conduct a pre-study about the way children touch with their fingers and present quantities on a table. The programming of the multi-touch-learning-environment is currently in progress using a multi-touch extension of a dynamic geometry software system.

Finally, we aim to answer the research question about the impact of the availability of such multi-touch-learning-environments regarding the diagnosis and the support of acquiring basic concepts of numbers and operations.

NOTES

1. MER: multiple external representations (Ainsworth, 1999)

2. MELRs: multiple equivalent linked representations (Harrop, 1999)

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