IMPLEMENTATION OF A MULTI-TOUCH-ENVIRONMENT SUPPORTING FINGER SYMBOL SETS

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Basic concepts of numbers and operations are fundamental for mathematical learning. Suitable materials for developing such basic concepts are hands and fingers. Among other things, this is because of their natural structure of 5 and 10. To support the development of concepts and the process of internalization a linking between different forms of representations by the computer can be helpful. To benefit of both, the advantages of the hands and fingers and the automatically linking, we suggest using multi-touch-technology, i.e. computer input devices that are able to recognize several touch gestures at the same time. Here, children can present numbers with their fingers that produce virtual objects. These objects can be automatically linked with the symbolic form of representation.

THE ORDINAL AND CARDINAL CONCEPT OF NUMBERS AND OPERATIONS

"How many things are there?" – For parents as well as for mathematicians, this is a common question to pose, if a child already has knowledge about numbers. For the child, this question is almost always the initiation to start counting verbally by saying the number words in a row (Fuson, 1988). The fundamental principles needed for answering the question are a) the one-one-principle that relates every single object to exactly one numeral (Gelmann & Gallistel, 1978), b) the stable-order-principle prescribing the correct order of numbers (Fig. 1, left), and c) the last-word-rule that assigns the last said numeral not the last counted object, but to the quantity as a whole (Fig. 1, right).

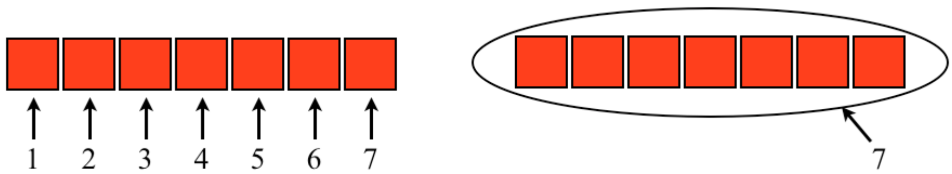

Figure 1: ordinal (left) and cardinal (right) concept of numbers

Here, the change from the ordinal concept of numbers, where the numeral is part of the numeral row, to the cardinal concept of numbers, where the numeral identifies a quantity, is necessary. It is not necessary to count a quantity in order to know it, that is, the ordinal concept is not a necessity for the cardinal concept. Resnick (1991) distinguishes the development of mathematical knowledge by two components that are developed independently: *protoquantitative schemata* and the *mental number line*. To build-up a well-developed concept of numbers, these two threads of development have to be linked. For many children this is a critical problem (Fuson, 1992 p. 63).

Children who do not have a proper linking between the two concepts can misinterpret addition and subtraction as a demand to count forwards or backwards. As long as the children calculate with numbers smaller than 20 they can apply this strategy successfully. But, for instance, when they want to add 55 to 27 and begin to count *„28, 29, 30, 31, ...“* there is no chance to come easily and quickly to the correct result.

„The protoquantitative part-whole schema is the foundation for later understanding of binary addition and subtraction and for several fundamental mathematical principles, such as the commutativity and associativity of addition and the complementarity of addition and subtraction. It also provides the framework for a concept of additive composition of number that underlies the place value system.“ (Resnick, 1991 p. 32).

For example when you want to add 6 and 8 with the use of the part-whole schema you can split and add in lots of ways (e.g. fig. 2).

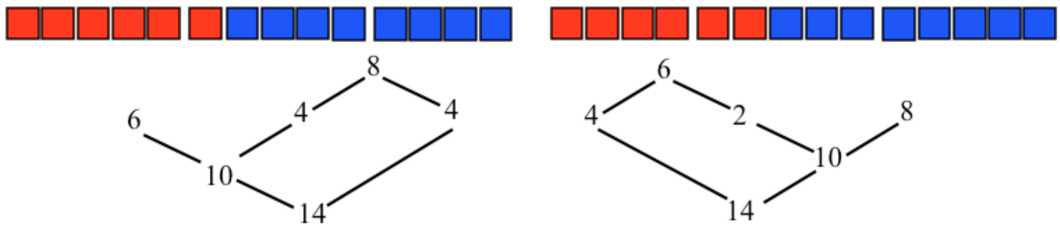

Figure 2: different ways to add with the part-whole-schema

FINGER SYMBOL SETS

Calculating with fingers has a very bad reputation in mathematics lessons, as it is usually seen as an indicator for counting. Most children do as they have learned from young days on and count objects by „Counting-Word Tagging to Number“ (Brissiaud, 1992). According to the ordinal concept of numbers each finger is related to exactly one numeral. To illustrate this we ask what happens if the sixth finger is buckled? The „name“ of the last finger, that indicated the quantity, was „10“ before, but now the finger has to be renamed into „9“ (Fig. 3).

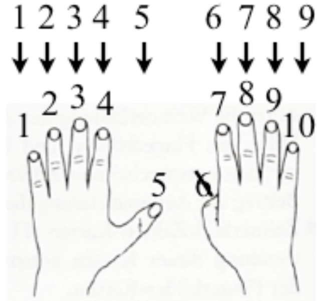

Figure 3: Order-irrelevance principle

The child has to know that it is irrelevant which fingers it uses to present a quantity.

To present „3“, the thumb, the index finger and the middle finger can be used as well as the little finger, the middle finger and the thumb, or any other combination of three fingers. As we point out below, the cognitive process behind this fact can be experienced and thus supported by the use of multi-touch-technology.

Amongst others, the advantages of fingers and hands are their permanent availability and their natural structure in *10* fingers per child with *5* fingers per hand. The 10 fingers qualify the hands to work out questions about the decimal number system, e.g. *„How many children do we need to see 30 fingers all at once?“* The „power of five“ (Krauthausen, 1995) is due to the ability to instantaneously recognize quantities (subitizing) up to 4. Applying this to the hands, the shown quantity of the fingers of one hand can be conceived simultaneously and hence the fingers of both hands can be conceived quasi-simultaneously. Furthermore, one hand gets a special status because children tend to present numbers greater than five sequentially (Brissiaud, 1992 p. 61). For example, to present „7“, they tend to use one full hand and then add two fingers of the other hand. In this way the decomposition of the numbers from 1 to 10 with the power of five can be worked out. But not only these, also all other decompositions are possible (Fig. 4) and can be conceived quasi-simultaneously.

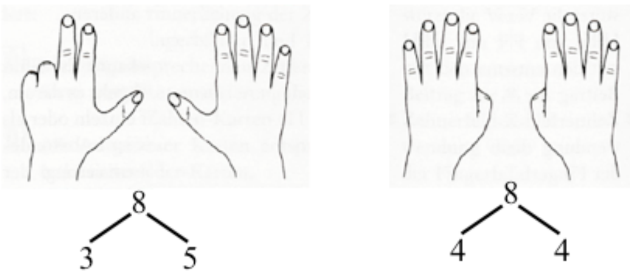

Figure 4: Decomposition of numbers with finger-symbol-sets

If the fingers are used like this, in sense of the part-whole schema, they are a qualified working material for a well-developed concept of numbers and operations (cf. Steinweg, 2009). Brissiaud (1992) coined the notion „From Finger Symbol Sets to Number“:

„Certain children who were not exposed early to the use of finger symbol sets may become counters, whereas children who were encouraged to use finger symbol sets may preferentially choose finger strategies“.

If children have a part-whole schema of numbers the transition to addition and subtraction is easy. It is just another way of nonverbal-symbolic representation of the fact that „two parts make a whole“.

Further strategies like variation in the opposite or in the same direction can than be worked out easily: If one finger is buckled, than another finger must be stretched to keep the same quantity. To get the difference of two quantities, e.g. of 9 and 7, you can vary the numbers in the same direction. For example, a whole hand can be omitted, which corresponds to subtracting five from each quantity. It is evident that the difference of 9 and 7 is the as the difference between 4 and 2. Based on such strategies the decadic analogy can be build up.

It is important to pay attention to the fact that the children stretch their fingers *simultaneously* to represent quantities with them. If they show them one-by-one the positive effects of these strategies are lost and the children will still use counting for addition and subtraction.

This introduction can only serve as a small insight into the possible representations of numbers and operations by hands and fingers and their usage in early arithmetic. It is the process of internalization that is of essential importance: How can the children benefit from the mathematical content of these representations and actions and use them in their mental processes?

THE PROCESS OF INTERNALIZATION SUPPORTED BY THE USE OF MULTI-TOUCH-TECHNOLOGY

According to Aebli, the process of early mathematical learning follows four stages, independent of the arithmetical subject (Grissemann & Weber, 2000; Aebli, 1987). Coming from concrete manipulations with different objects (stage 1), the children have to abstract these manipulations and operations to pictorial representations (stage 2). Subsequently they pass over to symbols (stage 3) with the aim to automate their actions (stage 4). For us, stage 2 is of special importance, because there the process of internalization takes place. The child has to comprehend the manipulation of concrete objects as a representation of a quantitative structure and it has to capture the structure and the relations of the concrete manipulation in an intellectual activity (Gerster & Schultz, 2004 p. 47). Lorenz calls this process „focus of attention". To facilitate this process of focus and abstraction and to develop it, a dialog is essential (Lorenz, 1997): *„In talking about the working material and the relations between numbers and operations that it represents, the concepts in development of the learner are going to be clarified by verbalisation."* In this sense, Aebli (1987) suggests that the children should review their concrete manipulations and make forecasts about further actions. Doing this, they comment their own manipulations by iconic illustrations till they are able to reproduce the structures and relations of the manipulations in conceptions. To support this process Aebli (1987 p. 238) established the following rule:

„Every new, more symbolic representation of the operation must be linked as closely as possible with the precedent one."

The enactive form of representation with finger symbol sets should be related to the nonverbal-symbolical form of representation (MER[1]) (Ainsworth, 1995; Mayer, 2005). But as studies show some of the children even don't link the different forms of representations when they are designed in form of MERs (Clements, 2002). For them, an automatic linking designed with the computer (MELRs[2]) can help them to experience the relations (Thompson, 1992; Clements, 2002; Ladel, 2009). This experience should be as natural and directly as possible. In this article we suggest to use multi-touch-technology for this experience, where the children can manipulate with their hands and fingers and an automatic linking with all other forms of representation can take place. In the remainder of this article we assume the availability of a multi-touch-enabled table. Such a table consists of a display surface connected to a computer and some tracking hardware that can recognize several touches on the display simultaneously and report them to the computer software. Similar technology with a different form factor is available in desktop monitors, tablet computers and devices like the Apple iPad, or mobile phones. With the availability of hardware as already imagined by Kay (1972) we now have to answer the question of the educational implications more than ever.

The basic underlying idea for all the activities sketched only briefly in the following is that the computer can track the children's actions on the table and give nonverbal-symbolic representations of either the current situation or the action that lead to it in form of a written protocol.

In a first scenario, the children represent numbers with their hands and fingers as described before. This enactive form of representation shall produce an iconic one on the display. The computer creates quadratic pads on the surface of the multi-touch-table. Through the contact of the fingers with the multi-touch-interface there is not only a link between the enactive form of representation with other forms of representation but also between the tactile and the visual sense. While representing numbers enactively and thus iconically, there is an automatic link to a nonverbal-symbolic form of representation. This representation can be imagined like a paper tape or sales slip and serves as a kind of protocol for the manipulations the children do. Such a protocol can support the focus of attention and the numerical aspects of a task (Dörfler, 1986).

In this activity it is possible for children to experience that it is of no particular importance which fingers they use to present quantities. At a table, it is also possible that the children work in teams: Two children can "share the work" to present two fingers if each touches the table with one finger. While this sounds funny for the number two, it is of great importance for partitions of larger numbers. Two partners can try to find all ways to partition numbers up to 20 into two numbers up to 10.

Working in teams or groups the children are also able to present numbers greater than 10, emphasizing the social aspects of learning. Because the protocol immediately reflects the actions of the children their focus of attention is on the mathematical content of their actions automatically, guiding them to abstraction.

It is also possible to support the four basic arithmetic operations and their basic concepts in such an environment. Regarding addition, students can develop the basic concept of a union by manipulating the virtual objects (pads) and arrange them close to each other. For example, if the child merges a group of 3 pads and a group of 5 pads the protocol will show the symbolic representation of this action as „3 + 5 = 8".

Here the focus of attention lies on the fact that this action constitutes a basic concept of addition, together with its nonverbal-symbolic form of representation. In multi-touch-technology there is also the possibility to draw a circle around some pads with the effect that these pads are bundled (a so-called lasso-gesture). This again is a manipulation based on the basic concept of union. Another task in the realm of addition and subtraction may be that 3 pads are shown and the child should create so many pads that in the end there are 7 (3 + _ = 7).

The basic concept of balance can be represented as well. Children can create quantities, remove from them, manipulate them with their fingers, and see the consequences of the manipulation at the same time in the nonverbal-symbolic protocol. Likewise it is possible to give instructions in the nonverbal-symbolic form and to see the output in the iconic forms with the pads.

It is rather easy to imagine that addition and subtraction can be done in such an environment, and we have shown some ways how the action or the state can be linked to a nonverbal-symbolic representation. For multiplication and division it is advisable to take advantage of the time as another dimension. The temporal-successive idea of multiplication that can be traced back to a repeated addition is mapped to a repeated touch action of the same quantity of fingers several times. The protocol may then show, for four touches with five fingers, "5 + 5 + 5 + 5 = 20" as well as "4 • 5 = 20". Thus the children can see, that there are different ways to protocol their manipulation. If several children are working together they can take advantage of the spatial-simultaneous idea of multiplication, creating the same quantity by several children at the same time. For division, one example activity would be to move pads and build piles of the same amount to divide a given number of pads.

TECHNOLOGICAL IMPLEMENTATION

In order to implement prototypical environments and for recording experimental data of children's interaction with a multi-touch enabled screen we used the interactive geometry software Cinderella (Richter-Gebert & Kortenkamp, 2006), which acts as a standard tool for rapid prototyping of learning environments. The customization of the learning environments is done via the integrated scripting language CindyScript (Richter-Gebert & Kortenkamp, 2010). CindyScript is a functional programming language that was designed to match standard mathematical expressions as closely as possible, while still providing all the structural elements of imperative programming.

As CindyScript can be triggered by user actions (like pressing the mouse or a key, moving the mouse, or starting a simulation) it is possible to change the standard behaviour of an IGS into the required interaction for an experiment.

A striking example for such a change in user interface behaviour is the method of adding points in the DOPPELMOPPEL learning environment (Ladel & Kortenkamp, 2009). Here, instead of having a dedicated mode to create points, points can be created in drag mode by pulling them from a never-ending stock onto a virtual table, and they can be deleted by just moving them off the table. For finger-symbol sets we adapt this technique: Pads can be created without referring to a stock pile, but just by placing fingers in an area next to the table. This allows for multiple pads to be created simultaneously, as is necessary for quasi-simultaneous representations of numbers.

This modeless operation of the learning environment (see Raskin (2000) for a discussion of modal operations in software) is necessary for any multi-touch environment: As one of the goals is the collaboration of several children, and the actions of the children cannot be differentiated, i.e., the computer cannot know which child is associated to which touch event, any mode would have to globally valid for all children at the same time. Switching to another mode (for example, switching between dragging pads and creating pads) would have to be announced and negotiated. Such negotiation would introduce too many obstacles in the user interaction and counters the collaborative advantages of multi-touch.

The latest version of Cinderella offers multi-touch support by adding the TUIO protocol for input events (Kortenkamp & Dohrmann, 2009). Currently, this support is restricted to allowing several elements to be dragged at the same time. Other modes, like the add-point mode or the add-line mode, are not multi-touch enabled. For the modeless operation as pointed out above we are using a helpful extension of the scripting facilities of Cinderella: Touch events (finger detected, finger moved, finger released) are translated into mouse events (mouse down, mouse move, mouse up). Using CindyScript, custom actions can be added to these touch events as it is possible with mouse events.

It is not straightforward to adapt a scripted interface to the fact that several press-drag-release sequences can happen simultaneously. It is customary to program user interfaces under the assumption that mouse events are exclusively delivered in the prescribed order of press, drag (possibly repeated), release. This is relevant for example if a program assumes a "currently moved element", like a currently moving point in an IGS. Designing software without that general assumption is much more difficult as it involves keeping track of all the current objects and states and their association to the touching fingers. CindyScript facilitates this design process by offering *touch-local-variables*: Declaring a variable to be touch-local using the `mtlocal()`-function assures the availability of a different instance of that variable for each press-drag-release sequence of a finger. This is very similar to the concept of switching contexts in recursive programs.

As an example, consider a program that will record the current mouse position in the mouse down event and then connect that position with the current mouse position in the drag event. A simple CindyScript implementation would be to place `start=mouse().xy;` in the mouse down event and `draw(start,mouse().xy);` in the drag event. Without declaring the `start` variable touch-local this would fail with multi-touch events, with the declaration `mtlocal(start);` it will work flawlessly with any number of simultaneous touches by recording the start position of each finger separately.

The exchange of global information (like the number of total touches) is easily possible by *not* declaring variables touch-local. Placing the commands `count=count+1;` and `count=count-1;` into the mouse down resp. mouse up events will keep track of the current number of fingers touching the surface.

We found the prototyping facilities of CindyScript with the touch extension to be very appropriate for our needs. The final behaviour of the learning environments is not yet determined and should be easily adaptable to empirical findings during the process of interface design. Also, any professional software programming services would need a full specification and, besides being to expensive in this early research stage, could not reflect the didactic considerations as described above.

FORECAST

We are currently working on implementing the above scenarios using a multi-touch-table built at CERMAT. A first study that examines the critical point in translating numbers and operations from and in different forms of representation has taken place in October 2010. At the same time we conducted a pre-study about the way children touch with their fingers and present quantities on a table.

Finally, we aim to answer the research question about the impact of the availability of such multi-touch-learning-environments regarding the diagnosis and the support of acquiring basic concepts of numbers and operations.

NOTES

1. MER: multiple external representations (Ainsworth, 1999)

2. MELRs: multiple equivalent linked representations (Harrop, 1999)

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